M E T U
Northern Cyprus Campus

	Calculus for Functions of Several Variables Short Exam 3	

Code : *Math 120* Acad.Year: *2011-2012* Semester : *Fall* Date : *09.12.2011* Time : *13:45* Duration : *55 minutes*	Last Name: Name: Department: Student No: Section: Signature: 3 QUESTIONS ON 2 PAGES TOTAL 32 POINTS

1	2	3			

Show your work! No calculators! Please draw a $\boxed{\text{box}}$ around your answers!

Please do not write on your desk!

1. (8 pts.) Calculate $\displaystyle\iint_R \frac{x}{x^2+y^2}\,dA$ where $R=[1,3]\times[0,4]$.

$$I=\int_0^4\int_1^3\frac{x}{x^2+y^2}\,dx\,dy=\int_0^4\int_1^3\frac12\frac{du}{u}\,dy=\frac12\int_0^4\ln(x^2+y^2)\,dy=\frac12\left(\int_0^4\ln(y^2+3^2)dy-\int_0^4\ln(y^2+1)dy\right)$$

$x^2+y^2=u$

$2x\,dx=du$

$$\int\ln(y^2+a^2)\,dy = y\cdot\ln(y^2+a^2)-\int\frac{2y^2}{y^2+a^2}\,dy$$

$u=\ln(y^2+a^2)$, $dv=dy$

$du=\frac{2y}{y^2+a^2}$, $v=y$

$$=y\ln(y^2+a^2)-\underbrace{\int 2\,dy}_{2y}+2a^2\underbrace{\int\frac{dy}{y^2+a^2}}_{J}$$

$$J=\int\frac{dy}{a^2\left(\left(\frac{y}{a}\right)^2+1\right)}=\frac{1}{a^2}\int\frac{a\,du}{u^2+1}=\frac1a\arctan u=\frac1a\arctan(y/a)$$

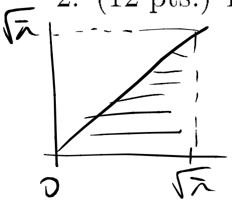
$u=y/a$

$du=dy/a$

$$I=\frac12\left[\left.y\ln(y^2+3^2)-2y+2\cdot3^2\cdot\frac13\arctan(y/3)\right|_0^4-\left.\left(y\ln(y^2+1)-2y+2\cdot1^2\cdot\frac11\arctan(y)\right)\right|_0^4\right]=\frac12\left[4\ln(25)+6\arctan(4/3)-\left(4\ln(17)+2\arctan 4\right)\right]$$

$$I=2\ln(25)+3\arctan(4/3)-2\ln(17)+\arctan 4\,.$$

2. (12 pts.) Evaluate the integral by reversing the order of integration.



$$\int_0^{\sqrt{\pi}}\int_y^{\sqrt{\pi}}\sin(x^2)\,dx\,dy=\int_0^{\sqrt{\pi}}\int_0^x\sin(x^2)\,dy\,dx$$

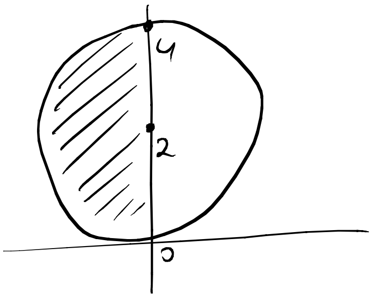
$$=\int_0^{\sqrt{\pi}}x\sin(x^2)\,dx=\int_0^{\pi}\sin u\,\frac{du}{2}$$

$\boxed{u=x^2,\quad du=2x\,dx}$

$$=\frac12\left.(-\cos u)\right|_0^{\pi}$$

$$=\frac{-1}{2}\left((-1)-0\right)=1/2\,.$$

3. (12 pts.) Evaluate the iterated integral by converting to polar coordinates.



$$I = \int_0^4 \int_{-\sqrt{4y-y^2}}^{0} \sqrt{x^2+y^2}\,dx\,dy$$

$x = -\sqrt{4y-y^2}$ $\quad x \le 0$

$x^2 = 4y - y^2$

$x^2 + y^2 - 4y + 4 = 4$

$x^2 + (y-2)^2 = 2^2$

$x^2 + y^2 = 4y$

$r^2 = 4r\sin\theta$

$r = 4\sin\theta$

In polar coordinates

$$I = \int_{\pi/2}^{\pi} \int_0^{4\sin\theta} \underbrace{\sqrt{r^2}\; r}_{r^2}\,dr\,d\theta$$

$$= \int_{\pi/2}^{\pi} \frac{r^3}{3}\Big|_0^{4\sin\theta} d\theta = \frac{64}{3}\int_{\frac{\pi}{2}}^{\pi} \sin^3\theta\,d\theta$$

$u = \sin^2\theta$ $\qquad dv = \sin\theta\,d\theta$

$du = 2\sin\theta\cos\theta$ $\qquad v = -\cos\theta$

$$= \frac{64}{3}\left[\underbrace{-\sin^2\theta\cos\theta\Big|_{\pi/2}^{\pi}}_{0} + 2\int_{\pi/2}^{\pi} \sin\theta\cos^2\theta\,d\theta\right]$$

$p = \cos\theta$

$dp = -\sin\theta\,d\theta$

$$= \frac{128}{3}\int_0^{-1} -p^2\,dp$$

$$= \frac{128}{3}\int_{-1}^{0} p^2\,dp = \frac{128}{3}\cdot\frac{p^3}{3}\Big|_{-1}^{0} = \frac{128}{9}\left(0 - (-1)\right)$$

$$= \frac{128}{9}$$