M E T U

Northern Cyprus Campus

Math 120	**Calculus for functions of several variables**	**Final Exam**	**05.06.2009**
Last Name : Name : Student No :	Dept./Sec. : Time : 09 : 30 Duration : 120 *minutes*	Signature	
7 QUESTIONS ON 4 PAGES		TOTAL 100 POINTS	

1	2	3	4	5	6	7			

Q1 (15=6+9 pts.) Test whether the following series converge or diverge, and explain your answer.

(a) $\sum_{n=1}^{\infty} \dfrac{(-1)^n\, n}{n+1}$

(b) $\sum_{n=1}^{\infty} \dfrac{\ln(n)}{n^2+2n-1}$

Q2 (15 pts.) Using Analytic Geometry, find the surface area of the tetrahedron bounded by the plane $2x + 3y + 4z = 24$ and the coordinate planes in the first octant.

Q3 (10 pts) Find the line integral $\oint_C \mathbf{F} \cdot \mathbf{dr}$, where

$$\mathbf{F}(x, y) = (\sin(y) - y\sin(x))\,\mathbf{i} + (\cos(x) + x\cos(y))\,\mathbf{j}$$

is the vector field and C is the path parametrized as $\mathbf{r}(t) = e^t \sin(t)\,\mathbf{i} + e^t \cos(t)\,\mathbf{j}$, $0 \leq t \leq 4\pi$.

Q4 (15=5+8+2 pts.) Let $f(x, y) = xy$ be a function defined on the region $x^2 + 4y^2 \leq 16$.

(a) Find and classify all critical points of the function f over the interior region $x^2 + 4y^2 < 16$.

(b) Use the method of Lagrange multipliers to find the maximum and minimum values of the function f over the boundary $x^2 + 4y^2 = 16$.

(c) Find the absolute max-min values of the function f over the region $x^2 + 4y^2 \leq 16$.

Q5 (15 pts.) Find the double integral

$$\int\int_R \frac{x+2y}{(2x-y)^2}dxdy$$

over the parallelogram R enclosed by the lines $x + 2y = 3$, $x + 2y = 5$, $2x - y = -6$ and $2x - y = -3$ (*Hint: Use the linear transformation* $u = x + 2y$, $v = 2x - y$*)*

Q6 (15 pts.) Express the volume enclosed by the surfaces $y = x^2 - 1$, $y = 0$, $z = x + y + 10$ and $z = -x - y - 4$ as a triple integal and evaluate this integral.

Q7 (15 pts.) Find $\oint_C (2y - \sin(\sin(x)))\,dx + (x - 2xy + \cos(\cos(y)))\,dy$, where C is the circle $x^2 + y^2 - 4y = 0$ oriented counterclockwise.