METU

Northern Cyprus Campus

Math 210	Applied Mathematics for Engineers	Lab Final	03.06.2010
Last Name : Name : Student No: SOLUTION	Dept./Sec. : Time : 16:40 Duration : 110 *minutes*	Signature	
4 QUESTIONS ON 4 PAGES		TOTAL 100 POINTS	
1	2	3	4

For applied questions, please write down all commands (or the content of the M-file) that you use in Matlab in order to perform the given task. You do not need to write anything about unsuccessful trials. Also write down the answer to the question. You may use Matlab's built in help whenever necessary; however you may not use any online resources (e.g. files on the course website). Opening any web-browser windows will lead to an automatic 0 on the exam.

25pts **Q1 (Applied)** The equation $f(x) = x - 5\cos(x)$ has three real roots.

(a) Write matlab code using a for loop iterating Newton's method 1000 times to find a root starting at the initial guess $x = 1$.

```
>> f= inline ('x-5*cos(x)', 'x')
>> df = inline (1+5*sin(x)', 'x')
>> x =1
>> for (iteration = 1:1000)
>> x= x- f(x)/ df(x) ;
>> end
```

10

Result: x= 1.3064 5

(b) Use Newton's method to find all of the other roots. (You do not need to rewrite the code from part (a) – just your initial guesses and the roots.) Check that your roots are correct.

Initial guess	Root	x-5*cos(x)
1	1.3064	0
-1	-1.9774	0
-4	-3.8375	0

10

25pts Q2 **(Applied)** Create a matlab variable a with the value .00001, and matrix

$$A = \begin{bmatrix} a & 1 & -a & 1 \\ 1 & -a & a & 1 \\ 2 & 1 & 1 & 4 \end{bmatrix}.$$

Write matlab code reducing A to row echelon form **using partial pivoting** (do not use any loops).

```
>> a = .00001
>> A = [a 1 -a 1; 1 -a a 1; 2 1 1 4]
>> A([1 3],:) = A([3 1],:)
>> A(2,:) = A(2,:) - A(1,:)*1/2
>> A(3,:) = A(3,:) - A(1,:)*a/2      (not required)
>> A([2 3],:) = A([3 2],:)
>> A(3,:) = A(3,:) + A(2,:)*.5/1
```

15 pts.

Result: $A = \begin{bmatrix} 2.0000 & 1.0000 & 1.0000 & 4.0000 \\ 0 & 1.0000 & -0.0000 & 1.0000 \\ 0 & -0.0000 & -0.5000 & -0.5000 \end{bmatrix}$

Note that the matrix A above is the augmented matrix for the system $\begin{bmatrix} a & 1 & -a \\ 1 & -a & a \\ 2 & 1 & 1 \end{bmatrix} x = \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}$

Use your reduced form of A to give the solution to this system without further use of matlab.

$-.5x_3 = -.5 \implies x_3 = 1$

$1x_2 - 0x_3 = 1 \implies x_2 = 1$

$2x_1 + 1x_2 + 1x_3 = 4 \implies x_1 = 1$

10 pts

$$\vec{x} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

25 pts

Q3 (Theory) Suppose the matrix A has LU decomposition

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix} \quad \text{and} \quad U = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

Use the LU decomposition to solve $Ax = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$ working by hand.

(Show your work. Do not use matlab. Do not compute A.)

THEORY REMINDER:

If $A\bar{x} = \bar{b}$, and $A = LU$ then

$$\left.\begin{array}{l} L U \bar{x} = \bar{b} \\ L(U\bar{x}) = \bar{b} \end{array}\right\} \text{5 pts.}$$

- First find $\bar{y}$ so that $L\bar{y} = \bar{b}$:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & -1 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

$$\Longrightarrow \quad \begin{array}{l} y_1 = 2 \\ y_2 = -1 \\ 2y_1 - y_2 + y_3 = 1 \Rightarrow y_3 = -4 \end{array} \qquad \boxed{\bar{y} = \begin{pmatrix} 2 \\ -1 \\ -4 \end{pmatrix}}$$

10 pts.

- Next find $\bar{x}$ so that $U\bar{x} = \bar{y}$:

$$\begin{pmatrix} -1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ -4 \end{pmatrix}$$

10 pts.

$$\Longrightarrow \quad \begin{array}{l} -x_1 + x_3 = 2 \Longrightarrow x_1 = -4 \\ x_2 + x_3 = -1 \Longrightarrow x_2 = 1 \\ 2x_3 = -4 \Longrightarrow x_3 = -2 \end{array} \qquad \boxed{\bar{x} = \begin{pmatrix} -4 \\ 1 \\ -2 \end{pmatrix}}$$

25 pts. Q4 (Applied) Use matlab to estimate the path integral $\int_C x^2y\,dx + y\,dy$ where C is the curve

$$C = \{\mathbf{r}(t) = \cos(2t)\mathbf{i} + \sin(2t)\mathbf{j} \cdot \text{for } \pi \le t \le 3\pi\}$$

METHOD 1

```
>> x= inline ('cos (2*t)', 'x')
>> y= inline ('sin (2*t)', 'y')
>> dt = (3*pi - pi)/1000
>> t_endpoints = pi: dt: 3*pi ;
>> t_centers = pi+ dt/2 : dt : 3*pi - dt/2 ;
>> dx = x(t_endpoints (2:end)) - x (t_endpoints (1:end-1)) ;
>> dy= y (t_endpoints (2:end)) - y(t_endpoints (1: end-1)) ;
>> integral = sum ( x(t_centers).^2.*y(t_centers).* dx ...
        + y(t_centers).*dy )
```

Result: integral = -1.5708

$$\int_c x^2y\,dx + y\,dy = \int_{\pi}^{3\pi} \cos^2 2t \sin 2t\,(-2\sin 2t) + \sin 2t\,(2\cos 2t)\,dt$$

$$= 2\int_{\pi}^{3\pi} \sin 2t \cos 2t - \cos^2 2t \sin^2 2t \; dt \qquad 5\text{pts.}$$

METHOD 2

```
>> dt = (3*pi - pi) / 1000
>> f= inline ('sin (2*t).*cos(2*t) - cos(2*t).^2.* sin(2*t).^2', 't')
>> t_centers = pi + dt/2 : dt : 3*pi - dt/2
>> integral = 2* sum ( f(t_centers). * dt)
```

20 pt.

Result: integral = -1.5708