Math 210 Applied Mathematics for Engineers I. Exam 24.03.2010

Last Name : Name : Student No:	Dept./Sec. : Time : 17: 40 Duration : 110 *minutes*	Signature

4 QUESTIONS ON 4 PAGES TOTAL 100 POINTS

1	2	3	4			

Q1 (5+5+5+5=20 pts.) Let

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 0 & -1 \\ 1 & 1 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix}, \qquad \mathbf{C} = \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}.$$

Compute the following (if defined):

(i) $\mathbf{BA}$.

(ii) $2\mathbf{B} + \mathbf{C}$.

(iii) $\mathbf{A}^{\mathrm{T}}$.

(iv) $(\mathbf{AB})^{\mathrm{T}}$.

Q2 (5+5+5+5+5+5=30 pts.) Consider the matrix equation $\mathbf{Ax} = \mathbf{b}$ given by

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ 5 \end{bmatrix}.$$

(i) Write the matrix equation as a <u>system of equations</u> and as an <u>augmented matrix</u>.

(ii) Compute the determinant of the matrix $\mathbf{A}$ <u>using row and/or column expansion</u>.

(iii) Find the solution to the matrix equation <u>using row reduction</u>.

(iv) Find the inverse of the matrix $\mathbf{A}$ (by any method).

(v) Find the solution to the matrix equation <u>using the inverse matrix from *(iv)*</u>.

(vi) Find x_3 <u>using Cramer's rule</u>.

Q3 (6+6+6+6+6+6=36 pts.)

The matrix $\mathbf{A}$ reduces as indicated below:

$$\mathbf{A} = \begin{bmatrix} 2 & 1 & 3 & 2 \\ 4 & 1 & 8 & 5 \\ -2 & -3 & 1 & -2 \\ 8 & 2 & 16 & 16 \end{bmatrix} \longrightarrow \begin{bmatrix} 2 & 1 & 3 & 2 \\ 0 & -1 & 2 & 1 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(i) What is the rank of $\mathbf{A}$? Why?

(ii) What is the nullity of $\mathbf{A}$ (i.e. the dimension of the nullspace)? Why?

(iii) Are the rows of A linearly independant? Why?

(iv) Is there a $\mathbf{b}$ so that $\mathbf{Ax} = \mathbf{b}$ has no solution? Why?

(v) Give the solution to $\mathbf{Ax} = \mathbf{0}$.

(vi) $\mathbf{Ax} = \begin{bmatrix} 4 \\ 4 \\ -10 \\ 2 \end{bmatrix}$ has a solution $\mathbf{x} = \begin{bmatrix} 4 \\ 1 \\ -1 \\ -1 \end{bmatrix}$. Give another solution.

(Hint: this should not require algebra.)

Q4 (14 pts.) Find the determinant of the following matrix:

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 & 3 \\ 1 & 2 & 3 & 4 & 4 \\ 1 & 2 & 3 & 4 & 5 \end{bmatrix}$$