# M E T U
## Northern Cyprus Campus

| **Math 210** | **Applied mathematics for engineers** | **Final Exam** | **11.08.2009** |
|---|---|---|---|
| Last Name: <br> Name : <br> Student No: | Dept./Sec. : <br> Time : 17: 40 <br> Duration : 120 *minutes* | Signature | |

| 4 QUESTIONS ON 4 PAGES | | | | | TOTAL 100 POINTS | |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | | | |

**Q1 (6+7+6+6=25 pts.)** Suppose that an elastic membrane, with the shape of a unit disk centered at the origin, in the $x_1 - x_2$ plane is subjected to an elastic deformation taking $(x_1, x_2)$ to $T(x_1, x_2) = (5x_1 + 3x_2, 4x_1 + x_2)$.

(a) Show that $T$ is a linear transformation, and write the matrix corresponding to $T$ with respect to the basis $\{(1,0),(0,1)\}$.

(b) Find the eigenvalues and eigenvectors of $T$.

(c) Find the new shape of the membrane after the transformation, and draw it.

(d) Find a basis consisting of eigenvectors of $T$, and write the matrix corresponding to $T$ with respect to this basis.

**Q2 (13+12=25 pts.)** Let $C$ be the boundary of the triangle with vertices $(0,0),(1,0)$ and $(0,1)$, traversed counterclockwise.

(a) Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x,y) = (x, 2y + x)$, by directly parameterizing the path.

(b) Evaluate the same integral by using Green's theorem.

**Q3 (8+8+9 pts.)** (a) Show that $div(curl\mathbf{F}) = 0$ for any vector field $\mathbf{F}$ on $\mathbb{R}^3$.

(b) Use part (a) to show that $\mathbf{G}(x, y, z) = (xy, x + y, z)$ cannot be written as the curl of a vector field $\mathbf{F}$.

(c) Let $\mathbf{H}(x, y, z) = (-x, -y, 2z + x)$. Check that $div\mathbf{H} = 0$ and find a vector field $\mathbf{F}$ such that $\mathbf{H} = curl\mathbf{F}$ (Hint: There are many possible answers. Make some choices).

**Q4 (9+8+8=25 pts.)** Suppose that we want to apply Newton-Ralphson iteration to find the roots of the equation $f(x) = 0$. We start from $x_0$, and denote the resulting sequence by $x_1, x_2, x_3, \ldots$. Suppose that $f$ is differentiable everywhere.

(a) Show that if $x_0$ is a local minimum or maximum of $f$, then $x_1$ is undefined.

(b) Do there exist functions $f$ such that $x_1$ is defined, but $x_2$ is undefined? (Drawing graphs would be enough for giving examples).

(c) Can one have a "ping-pong effect"? Namely, can the sequence be $x_0, x_1, x_0, x_1, x_0, \ldots$ with $x_0 \neq x_1$?