M E T U
Northern Cyprus Campus

Math 210	Applied mathematics for engineers	I. Exam	18.07.2009
Last Name: Name : Student No:	Dept./Sec. : Time : 10: 30 Duration : 100 *minutes*	Signature	

4 QUESTIONS ON 4 PAGES						TOTAL 100 POINTS
1	2	3	4			

Q1 (25 pts.) Consider the system of equations

$$\begin{aligned} 2x &\quad +3y &\quad +az &= 1 \\ x &\quad +2y &\quad +0z &= b \\ 4x &\quad +3y &\quad +5z &= a. \end{aligned}$$

Find all solutions of the system. Please carefully distinguish the different possibilities depending on the values of a and b.

Q2 (25 pts.) Let

$$A = \begin{bmatrix} 1 & -2 & 4 \\ 2 & 0 & -2 \\ 5 & 1 & 2 \\ -4 & 0 & 2 \end{bmatrix}$$

Determine bases for the row space and the column space of A, and find the rank of A. Identify the row space as a subset of $\mathbb{R}^3$ and the column space as a subset of $\mathbb{R}^4$.

Q3 (5+5+5+5+5=25 pts.) Answer the following (independent) questions. Assume that A and B are $n \times n$ matrices. Give reasons (proofs or counterexamples) supporting your answers.

(a) Is it always true that $rank(A + B)$ is greater than or equal to at least one of $rank(A)$ or $rank(B)$?

(b) Consider the set of vectors $V = \{(v_1, v_2) \in \mathbb{R}^2 | |v_1 + v_2| < 10\}$. Is V a vector space?

(c) Suppose that V is a vector space, and $S \subset V$ is a linearly independent subset. Is every subset of S also linearly independent?

(d) True or false: If a linear system has more unknowns than equations, then it cannot have a unique solution.

(e) True or false: If a linear system has more equations than unknowns, then it cannot have a unique solution.

Q4 (10+15=25 pts.) (a) Evaluate the following determinant:

$$\begin{vmatrix} 3 & 0 & 1 & 0 \\ 0 & 5 & 0 & 1 \\ 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & -1 \end{vmatrix}.$$

(b) Find a formula (in terms of a_i, b_i) for the value of the following determinant (all the blank spaces are zeros) :

$$\begin{vmatrix} a_1 & & & & 1 & & & \\ & a_2 & & & & 1 & & \\ & & \cdots & & & & \cdots & \\ & & & a_n & & & & 1 \\ 1 & & & & b_1 & & & \\ & 1 & & & & b_2 & & \\ & & \cdots & & & & \cdots & \\ & & & 1 & & & & b_n \end{vmatrix}.$$