M E T U
Northern Cyprus Campus

Code : *Math 120* Acad.Year: *2011-2012* Semester : *Fall* Date : *22.11.2011* Time : *18:45* Duration : *45 minutes*	**Calculus for Functions of Several Variables** **Short Exam 2** Last Name: Department: Section: Name: Student No: Signature: 5 QUESTIONS ON 2 PAGES TOTAL 32 POINTS	

1	2	3	4			

Show your work! No calculators! Please draw a $\boxed{\textbf{box}}$ around your answers!

Please do not write on your desk!

1. ($2 \times 4 = 8$ pts.) Find the limit, if it exists and prove your claim. Otherwise, show that the limit does not exist.

   (a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3y}{x^4+y^4}$

   On $x=0$ $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3y}{x^4+y^4} = \lim_{y\to0}\frac{0}{y^4} = 0$

   On $x=y$ $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3y}{x^4+y^4} = \lim_{y\to0}\frac{y^4}{2y^4} = \frac{1}{2}$

   If the limit existed, it would be the same evaluated in every direction, hence the limit does not exist.

   (b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{-\sin^3(x^2+\pi)y^6}{x^4+3y^4}$

   Notice $\displaystyle -y^2 \le \frac{-\sin^3(x^2+\pi)y^6}{x^4+3y^4} \le y^2.$

   Since the limits of both sides as (x,y) goes to $(0,0)$ are 0, by Squeeze Theorem, the required limit exists and is equal to 0.

2. (4 pts.) Find all the partial derivatives of the function $u = y^{z/x}$.

   $\displaystyle\frac{\partial u}{\partial x} = y^{z/x}\ln(y)(-z/x^2), \quad \frac{\partial u}{\partial y} = (z/x)y^{z/x-1}, \quad \frac{\partial u}{\partial z} = y^{z/x}\ln(y)(1/x)$

3. (4 pts.) Use the Chain Rule to find $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$ if $z = xy^2, \quad x = s\cos^2(t), \quad y = s^3\sin(t)$.

   $\displaystyle\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s} = (y^2)(\cos^2(t)) + (2xy)(3s^2\sin(t))$

   $\displaystyle\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t} = (y^2)(-2s\cos(t)\sin(t)) + (2xy)(s^3\cos(t))$

4. (4 pts.) Find the directional derivative of $f(x,y,z) = \sqrt{x^2yz}$ at the point $(2,1,1)$ in the direction of the vector $\mathbf{v} = (0,4,3)$.

Unit vector $\mathbf{u}$ in the direction of $\mathbf{v} = (0, \frac{4}{5}, \frac{3}{5})$.
Then we are looking for $D_{\mathbf{u}}f(2,1,1) = \nabla f(2,1,1) \bullet \mathbf{u}$
$\nabla f = (\frac{2xyz}{2\sqrt{x^2yz}}, \frac{x^2z}{2\sqrt{x^2yz}}, \frac{x^2y}{2\sqrt{x^2yz}})$, $\quad \nabla f(2,1,1) = (1,1,1)$
Hence, $D_{\mathbf{u}}f(2,1,1) = 0 + \frac{4}{5} + \frac{3}{5} = \frac{7}{5}$

5. (12 pts.) Find the absolute maximum and minimum values of $f(x,y) = 4x + 2y - x^2 - y^2$ on the set $D = \{(x,y)|0 \le x \le 4, 0 \le y \le 3\}$.
The function f is continuous (polynomial) on a bounded and closed domain D. Hence the absolute maximum and minimum are attained.
$\nabla f = (4-2x, 2-2y) = (0,0)$ gives the only critical point as (2,1) and this point is in the rectangle D. Note that $f(2,1) = 5$
On $x = 0$, let $g(y) = f(0,y) = 2y - y^2, \quad 0 \le y \le 3$. Critical points of $g(y)$ are found by looking at $g'(y) = 2 - 2y = 0$, hence $y = 1$ is the only critical point of $g(y)$. The extremes of g can appear when $y = 0, y = 1$, and $y = 3$, and they are $g(0) = f(0,0) = 0, g(1) = f(0,1) = 1, g(3) = f(0,3) = -3$.

On $x = 4$, let $h(y) = f(4,y) = 2y - y^2, \quad 0 \le y \le 3$. Critical points of $h(y)$ are found by looking at $h'(y) = 2 - 2y = 0$, hence $y = 1$ is the only critical point of $h(y)$. The extremes of h can appear when $y = 0, y = 1$, and $y = 3$, and they are $h(0) = f(4,0) = 0, h(1) = f(4,1) = 1, h(3) = f(4,3) = -3$.

On $y = 0$, let $a(x) = f(x,0) = 4x - x^2, \quad 0 \le x \le 4$. $a'(x) = 4 - 2x = 0$, hence $x = 2$ is the only critical point of $a(x)$. The extremes of a can appear when $x = 0, x = 2$, and $x = 4$, and they are $a(0) = f(0,0) = 0, a(2) = f(2,0) = 4, a(4) = f(4,0) = 0$.

On $y = 3$, let $b(x) = f(x,3) = 4x - x^2 - 3, \quad 0 \le x \le 4$. $b'(x) = 4 - 2x = 0$, hence $x = 2$ is the only critical point of $b(x)$. The extremes of b can appear when $x = 0, x = 2$, and $x = 4$, and they are $b(0) = f(0,3) = -3, b(2) = f(2,3) = 1, b(4) = f(4,3) = -3$.

Finally, out of all the possible values for local extremes, one must be absolute maximum, one must be absolute minimum. Gazing around all the points we have mentioned above, we see that :

Absolute maximum of f is 5 attained at the interior point (2,1).
Absolute minimum of f is -3 attained at the boundary points (0,3) and (4,3).