M E T U
Northern Cyprus Campus

	Calculus for Functions of Several Variables **Final**	

Code : *Math 120* Acad.Year : *2009-2010* Semester : *Spring* Date : *4.6.2010* Time : *9:30* Duration : *120 minutes*	Last Name: Name : Student No: Department: Section: Signature:
	6 QUESTIONS ON 6 PAGES TOTAL 100 POINTS

1	2	3	4	5	6		

1 (4+12 pts.). Let $f(x) = 2^{-x}$ and A the total shaded area below.

(a) Express A as an infinite series.

(b) Does this series converge? If so, what is its value?

2.(16 pts.) Find the shortest distance between the plane $4x + 2y - z = 20$ and the paraboloid $z = x^2 + y^2$, using Lagrange Multipliers Method.

3.(16 pts.) Calculate the double integral

$$\int\int_R \left(12y^2 - 12xy - 24x^2\right)\, dx\, dy$$

over the parallelogram R bounded by the lines $y - 2x = 6$, $y - 2x = -4$, $y + x = 6$, and $y + x = 0$. (Hint: Use a change of variables.)

4.(6+10 pts.) Consider the iterated integral

$$\int_0^4 \int_{-\sqrt{4-z}}^{\sqrt{4-z}} \int_{-\sqrt{4-z-x^2}}^{\sqrt{4-z-x^2}} 1 \; dy \, dx \, dz$$

(a) Change the order of integration to $dz\,dx\,dy$.

(b) Change to cylindrical coordinates, and evaluate the triple integral.

5.(6+6+6 pts.) For each of the vector fields below, check whether it is conservative or not. Find a potential function if it is conservative.

(a) $\mathbf{F}(x, y, z) = \langle x^3, y^3, z^3 + 1 \rangle$.

(b) $\mathbf{F}(x, y, z) = \langle e^{x\cos(y)}, \tan(y)\, e^{x\cos(y)}(\sec(y) - x), xy \rangle$.

(c) $\mathbf{F}(x, y) = \langle (1 + xy)e^{xy}, e^y + x^2 e^{xy} \rangle$.

6.(6+12 pts.) (a) Show that

$$\oint_C (2xy + e^{x^2})dx + ((x+1)^2 + \ln(2 + \sin(y)))dy = 2\oint_C (x - y)dy$$

where C is the plane curve parametrized as $x = \cos(t) + \frac{1}{10}\cos^2(t)$, $x = \sin(t) + \frac{1}{10}\cos^2(t)$ for $0 \leq t \leq 2\pi$. (Hint: Use Green's theorem.)

(b) Evaluate the second line integral above directly using the parametrization.

(c) What is the area of the region enclosed by C ?