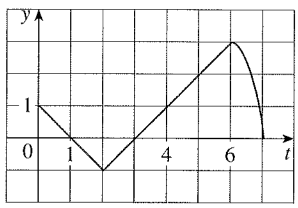
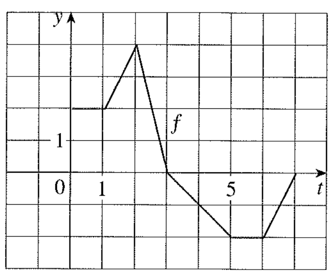
4.3 Exercises

1. Explain exactly what is meant by the statement that "differentiation and integration are inverse processes."

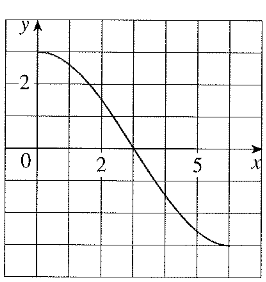
2. Let $g(x) = \int_0^x f(t)\,dt$, where f is the function whose graph is shown.
(a) Evaluate $g(x)$ for $x = 0, 1, 2, 3, 4, 5$, and 6.
(b) Estimate $g(7)$.
(c) Where does g have a maximum value? Where does it have a minimum value?
(d) Sketch a rough graph of g.



3. Let $g(x) = \int_0^x f(t)\,dt$, where f is the function whose graph is shown.
(a) Evaluate $g(0)$, $g(1)$, $g(2)$, $g(3)$, and $g(6)$.
(b) On what interval is g increasing?
(c) Where does g have a maximum value?
(d) Sketch a rough graph of g.



4. Let $g(x) = \int_0^x f(t)\,dt$, where f is the function whose graph is shown.
(a) Evaluate $g(0)$ and $g(6)$.
(b) Estimate $g(x)$ for $x = 1, 2, 3, 4$, and 5.
(c) On what interval is g increasing?
(d) Where does g have a maximum value?
(e) Sketch a rough graph of g.
(f) Use the graph in part (e) to sketch the graph of $g'(x)$. Compare with the graph of f.



5–6 Sketch the area represented by $g(x)$. Then find $g'(x)$ in two ways: (a) by using Part 1 of the Fundamental Theorem and (b) by evaluating the integral using Part 2 and then differentiating.

5. $g(x) = \int_1^x t^2\,dt$

6. $g(x) = \int_0^x (2 + \sin t)\,dt$

7–18 Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

7. $g(x) = \int_1^x \dfrac{1}{t^3 + 1}\,dt$

8. $g(x) = \int_1^x (2 + t^4)^5\,dt$

9. $g(s) = \int_5^s (t - t^2)^8\,dt$

10. $g(r) = \int_0^r \sqrt{x^2 + 4}\,dx$

11. $F(x) = \int_x^\pi \sqrt{1 + \sec t}\,dt$

$\left[\textit{Hint: } \int_x^\pi \sqrt{1 + \sec t}\,dt = -\int_\pi^x \sqrt{1 + \sec t}\,dt\right]$

12. $G(x) = \int_x^1 \cos\sqrt{t}\,dt$

13. $h(x) = \int_2^{1/x} \sin^4 t\,dt$

14. $h(x) = \int_1^{\sqrt{x}} \dfrac{z^2}{z^4 + 1}\,dz$

15. $y = \int_0^{\tan x} \sqrt{t + \sqrt{t}}\,dt$

16. $y = \int_0^{x^4} \cos^2\theta\,d\theta$

17. $y = \int_{1-3x}^1 \dfrac{u^3}{1 + u^2}\,du$

18. $y = \int_{\sin x}^1 \sqrt{1 + t^2}\,dt$

19–38 Evaluate the integral.

19. $\int_{-1}^2 (x^3 - 2x)\,dx$

20. $\int_{-1}^1 x^{100}\,dx$

21. $\int_1^4 (5 - 2t + 3t^2)\,dt$

22. $\int_0^1 \left(1 + \frac{1}{2}u^4 - \frac{2}{5}u^9\right)du$

23. $\int_0^1 x^{4/5}\,dx$

24. $\int_1^8 \sqrt[3]{x}\,dx$

25. $\int_1^2 \dfrac{3}{t^4}\,dt$

26. $\int_\pi^{2\pi} \cos\theta\,d\theta$

27. $\int_0^2 x(2 + x^5)\,dx$

28. $\int_0^1 \left(3 + x\sqrt{x}\right)dx$

29. $\int_1^9 \dfrac{x - 1}{\sqrt{x}}\,dx$

30. $\int_0^2 (y - 1)(2y + 1)\,dy$

31. $\int_0^{\pi/4} \sec^2 t\,dt$

32. $\int_0^{\pi/4} \sec\theta\tan\theta\,d\theta$

33. $\int_1^2 (1 + 2y)^2\,dy$

34. $\int_1^2 \dfrac{s^4 + 1}{s^2}\,ds$

35. $\displaystyle\int_1^2 \frac{v^5 + 3v^6}{v^4}\, dv$ **36.** $\displaystyle\int_1^{18} \sqrt{\frac{3}{z}}\, dz$

37. $\displaystyle\int_0^{\pi} f(x)\, dx$ where $f(x) = \begin{cases} \sin x & \text{if } 0 \leqslant x < \pi/2 \\ \cos x & \text{if } \pi/2 \leqslant x \leqslant \pi \end{cases}$

38. $\displaystyle\int_{-2}^{2} f(x)\, dx$ where $f(x) = \begin{cases} 2 & \text{if } -2 \leqslant x \leqslant 0 \\ 4 - x^2 & \text{if } 0 < x \leqslant 2 \end{cases}$

39–42 What is wrong with the equation?

39. $\displaystyle\int_{-2}^{1} x^{-4}\, dx = \frac{x^{-3}}{-3}\bigg]_{-2}^{1} = -\frac{3}{8}$

40. $\displaystyle\int_{-1}^{2} \frac{4}{x^3}\, dx = -\frac{2}{x^2}\bigg]_{-1}^{2} = \frac{3}{2}$

41. $\displaystyle\int_{\pi/3}^{\pi} \sec\theta \tan\theta\, d\theta = \sec\theta\Big]_{\pi/3}^{\pi} = -3$

42. $\displaystyle\int_0^{\pi} \sec^2 x\, dx = \tan x\Big]_0^{\pi} = 0$

43–46 Use a graph to give a rough estimate of the area of the region that lies beneath the given curve. Then find the exact area.

43. $y = \sqrt[3]{x}, \quad 0 \leqslant x \leqslant 27$ **44.** $y = x^{-4}, \quad 1 \leqslant x \leqslant 6$

45. $y = \sin x, \ 0 \leqslant x \leqslant \pi$ **46.** $y = \sec^2 x, \ 0 \leqslant x \leqslant \pi/3$

47–48 Evaluate the integral and interpret it as a difference of areas. Illustrate with a sketch.

47. $\displaystyle\int_{-1}^{2} x^3\, dx$ **48.** $\displaystyle\int_{\pi/6}^{2\pi} \cos x\, dx$

49–52 Find the derivative of the function.

49. $\displaystyle g(x) = \int_{2x}^{3x} \frac{u^2 - 1}{u^2 + 1}\, du$

$$\left[\textit{Hint: } \int_{2x}^{3x} f(u)\, du = \int_{2x}^{0} f(u)\, du + \int_0^{3x} f(u)\, du \right]$$

50. $\displaystyle g(x) = \int_{1-2x}^{1+2x} t \sin t\, dt$ **51.** $\displaystyle h(x) = \int_{\sqrt{x}}^{x^3} \cos(t^2)\, dt$

52. $\displaystyle g(x) = \int_{\tan x}^{x^2} \frac{1}{\sqrt{2 + t^4}}\, dt$

53. On what interval is the curve

$$y = \int_0^x \frac{t^2}{t^2 + t + 2}\, dt$$

concave downward?

54. If $f(x) = \int_0^x (1 - t^2)\cos^2 t\, dt$, on what interval is f increasing?

55. If $f(1) = 12$, f' is continuous, and $\int_1^4 f'(x)\, dx = 17$, what is the value of $f(4)$?

56. If $f(x) = \int_0^{\sin x} \sqrt{1 + t^2}\, dt$ and $g(y) = \int_3^y f(x)\, dx$, find $g''(\pi/6)$.

57. The Fresnel function S was defined in Example 3 and graphed in Figures 7 and 8.

(a) At what values of x does this function have local maximum values?

(b) On what intervals is the function concave upward?

CAS (c) Use a graph to solve the following equation correct to two decimal places:

$$\int_0^x \sin(\pi t^2/2)\, dt = 0.2$$

CAS **58.** The **sine integral function**

$$\operatorname{Si}(x) = \int_0^x \frac{\sin t}{t}\, dt$$

is important in electrical engineering. [The integrand $f(t) = (\sin t)/t$ is not defined when $t = 0$, but we know that its limit is 1 when $t \to 0$. So we define $f(0) = 1$ and this makes f a continuous function everywhere.]

(a) Draw the graph of Si.

(b) At what values of x does this function have local maximum values?

(c) Find the coordinates of the first inflection point to the right of the origin.

(d) Does this function have horizontal asymptotes?

(e) Solve the following equation correct to one decimal place:

$$\int_0^x \frac{\sin t}{t}\, dt = 1$$

59–60 Let $g(x) = \int_0^x f(t)\, dt$, where f is the function whose graph is shown.

(a) At what values of x do the local maximum and minimum values of g occur?

(b) Where does g attain its absolute maximum value?

(c) On what intervals is g concave downward?

(d) Sketch the graph of g.

59.