D. Since $x^2 + 1$ is never 0, there is no vertical asymptote. Since $f(x) \to \infty$ as $x \to \infty$ and $f(x) \to -\infty$ as $x \to -\infty$, there is no horizontal asymptote. But long division gives

$$f(x) = \frac{x^3}{x^2+1} = x - \frac{x}{x^2+1}$$

$$f(x) - x = -\frac{x}{x^2+1} = -\frac{\frac{1}{x}}{1 + \frac{1}{x^2}} \to 0 \quad \text{as} \quad x \to \pm\infty$$

So the line $y = x$ is a slant asymptote.

E.
$$f'(x) = \frac{3x^2(x^2+1) - x^3 \cdot 2x}{(x^2+1)^2} = \frac{x^2(x^2+3)}{(x^2+1)^2}$$

Since $f'(x) > 0$ for all x (except 0), f is increasing on $(-\infty, \infty)$.

F. Although $f'(0) = 0$, f' does not change sign at 0, so there is no local maximum or minimum.

G.
$$f''(x) = \frac{(4x^3 + 6x)(x^2+1)^2 - (x^4 + 3x^2) \cdot 2(x^2+1)2x}{(x^2+1)^4} = \frac{2x(3 - x^2)}{(x^2+1)^3}$$

Since $f''(x) = 0$ when $x = 0$ or $x = \pm\sqrt{3}$, we set up the following chart:

Interval	x	$3 - x^2$	$(x^2+1)^3$	$f''(x)$	f
$x < -\sqrt{3}$	$-$	$-$	$+$	$+$	CU on $(-\infty, -\sqrt{3})$
$-\sqrt{3} < x < 0$	$-$	$+$	$+$	$-$	CD on $(-\sqrt{3}, 0)$
$0 < x < \sqrt{3}$	$+$	$+$	$+$	$+$	CU on $(0, \sqrt{3})$
$x > \sqrt{3}$	$+$	$-$	$+$	$-$	CD on $(\sqrt{3}, \infty)$

The points of inflection are $(-\sqrt{3}, -\frac{3}{4}\sqrt{3})$, $(0, 0)$, and $(\sqrt{3}, \frac{3}{4}\sqrt{3})$.

H. The graph of f is sketched in Figure 11.

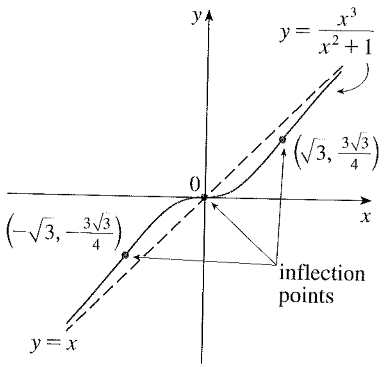


FIGURE 11

3.5 Exercises

1–40 Use the guidelines of this section to sketch the curve.

1. $y = x^3 + x$
2. $y = x^3 + 6x^2 + 9x$
3. $y = 2 - 15x + 9x^2 - x^3$
4. $y = 8x^2 - x^4$
5. $y = x(x-4)^3$
6. $y = x^5 - 5x$
7. $y = \frac{1}{5}x^5 - \frac{8}{3}x^3 + 16x$
8. $y = (4 - x^2)^5$
9. $y = \frac{x}{x-1}$
10. $y = \frac{x^2-4}{x^2-2x}$
11. $y = \frac{x - x^2}{2 - 3x + x^2}$
12. $y = \frac{x}{x^2-9}$
13. $y = \frac{1}{x^2-9}$
14. $y = \frac{x^2}{x^2+9}$

1. Homework Hints available at stewartcalculus.com

15. $y = \dfrac{x}{x^2 + 9}$

16. $y = 1 + \dfrac{1}{x} + \dfrac{1}{x^2}$

17. $y = \dfrac{x - 1}{x^2}$

18. $y = \dfrac{x}{x^3 - 1}$

19. $y = \dfrac{x^2}{x^2 + 3}$

20. $y = \dfrac{x^3}{x - 2}$

21. $y = (x - 3)\sqrt{x}$

22. $y = 2\sqrt{x} - x$

23. $y = \sqrt{x^2 + x - 2}$

24. $y = \sqrt{x^2 + x} - x$

25. $y = \dfrac{x}{\sqrt{x^2 + 1}}$

26. $y = x\sqrt{2 - x^2}$

27. $y = \dfrac{\sqrt{1 - x^2}}{x}$

28. $y = \dfrac{x}{\sqrt{x^2 - 1}}$

29. $y = x - 3x^{1/3}$

30. $y = x^{5/3} - 5x^{2/3}$

31. $y = \sqrt[3]{x^2 - 1}$

32. $y = \sqrt[3]{x^3 + 1}$

33. $y = \sin^3 x$

34. $y = x + \cos x$

35. $y = x \tan x, \quad -\pi/2 < x < \pi/2$

36. $y = 2x - \tan x, \quad -\pi/2 < x < \pi/2$

37. $y = \frac{1}{2}x - \sin x, \quad 0 < x < 3\pi$

38. $y = \sec x + \tan x, \quad 0 < x < \pi/2$

39. $y = \dfrac{\sin x}{1 + \cos x}$

40. $y = \dfrac{\sin x}{2 + \cos x}$

---

41. In the theory of relativity, the mass of a particle is

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

where m_0 is the rest mass of the particle, m is the mass when the particle moves with speed v relative to the observer, and c is the speed of light. Sketch the graph of m as a function of v.

42. In the theory of relativity, the energy of a particle is

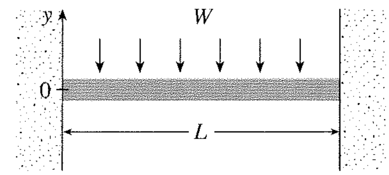
$$E = \sqrt{m_0^2 c^4 + h^2 c^2/\lambda^2}$$

where m_0 is the rest mass of the particle, λ is its wave length, and h is Planck's constant. Sketch the graph of E as a function of λ. What does the graph say about the energy?

43. The figure shows a beam of length L embedded in concrete walls. If a constant load W is distributed evenly along its length, the beam takes the shape of the deflection curve

$$y = -\frac{W}{24EI}x^4 + \frac{WL}{12EI}x^3 - \frac{WL^2}{24EI}x^2$$

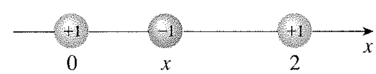
where E and I are positive constants. (E is Young's modulus of elasticity and I is the moment of inertia of a cross-section of the beam.) Sketch the graph of the deflection curve.



44. Coulomb's Law states that the force of attraction between two charged particles is directly proportional to the product of the charges and inversely proportional to the square of the distance between them. The figure shows particles with charge 1 located at positions 0 and 2 on a coordinate line and a particle with charge -1 at a position x between them. It follows from Coulomb's Law that the net force acting on the middle particle is

$$F(x) = -\frac{k}{x^2} + \frac{k}{(x - 2)^2} \qquad 0 < x < 2$$

where k is a positive constant. Sketch the graph of the net force function. What does the graph say about the force?



45–48 Find an equation of the slant asymptote. Do not sketch the curve.

45. $y = \dfrac{x^2 + 1}{x + 1}$

46. $y = \dfrac{2x^3 + x^2 + x + 3}{x^2 + 2x}$

47. $y = \dfrac{4x^3 - 2x^2 + 5}{2x^2 + x - 3}$

48. $y = \dfrac{5x^4 + x^2 + x}{x^3 - x^2 + 2}$

49–54 Use the guidelines of this section to sketch the curve. In guideline D find an equation of the slant asymptote.

49. $y = \dfrac{x^2}{x - 1}$

50. $y = \dfrac{1 + 5x - 2x^2}{x - 2}$

51. $y = \dfrac{x^3 + 4}{x^2}$

52. $y = \dfrac{x^3}{(x + 1)^2}$

53. $y = \dfrac{2x^3 + x^2 + 1}{x^2 + 1}$

54. $y = \dfrac{(x + 1)^3}{(x - 1)^2}$

---

55. Show that the curve $y = \sqrt{4x^2 + 9}$ has two slant asymptotes: $y = 2x$ and $y = -2x$. Use this fact to help sketch the curve.

56. Show that the curve $y = \sqrt{x^2 + 4x}$ has two slant asymptotes: $y = x + 2$ and $y = -x - 2$. Use this fact to help sketch the curve.

57. Show that the lines $y = (b/a)x$ and $y = -(b/a)x$ are slant asymptotes of the hyperbola $(x^2/a^2) - (y^2/b^2) = 1$.

58. Let $f(x) = (x^3 + 1)/x$. Show that

$$\lim_{x \to \pm\infty} [f(x) - x^2] = 0$$

This shows that the graph of f approaches the graph of $y = x^2$, and we say that the curve $y = f(x)$ is *asymptotic* to the parabola $y = x^2$. Use this fact to help sketch the graph of f.

59. Discuss the asymptotic behavior of $f(x) = (x^4 + 1)/x$ in the same manner as in Exercise 58. Then use your results to help sketch the graph of f.

60. Use the asymptotic behavior of $f(x) = \cos x + 1/x^2$ to sketch its graph without going through the curve-sketching procedure of this section.

3.6 Graphing with Calculus *and* Calculators

If you have not already read Appendix G, you should do so now. In particular, it explains how to avoid some of the pitfalls of graphing devices by choosing appropriate viewing rectangles.

The method we used to sketch curves in the preceding section was a culmination of much of our study of differential calculus. The graph was the final object that we produced. In this section our point of view is completely different. Here we *start* with a graph produced by a graphing calculator or computer and then we refine it. We use calculus to make sure that we reveal all the important aspects of the curve. And with the use of graphing devices we can tackle curves that would be far too complicated to consider without technology. The theme is the *interaction* between calculus and calculators.

EXAMPLE 1 Graph the polynomial $f(x) = 2x^6 + 3x^5 + 3x^3 - 2x^2$. Use the graphs of f' and f'' to estimate all maximum and minimum points and intervals of concavity.

SOLUTION If we specify a domain but not a range, many graphing devices will deduce a suitable range from the values computed. Figure 1 shows the plot from one such device if we specify that $-5 \leqslant x \leqslant 5$. Although this viewing rectangle is useful for showing that the asymptotic behavior (or end behavior) is the same as for $y = 2x^6$, it is obviously hiding some finer detail. So we change to the viewing rectangle $[-3, 2]$ by $[-50, 100]$ shown in Figure 2.

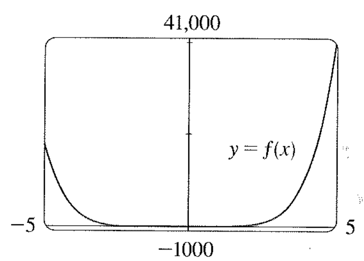


FIGURE 1

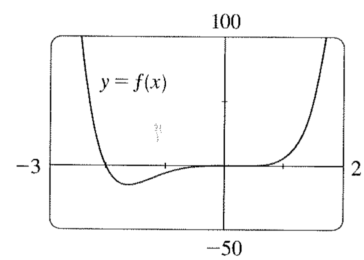


FIGURE 2

From this graph it appears that there is an absolute minimum value of about -15.33 when $x \approx -1.62$ (by using the cursor) and f is decreasing on $(-\infty, -1.62)$ and increasing on $(-1.62, \infty)$. Also there appears to be a horizontal tangent at the origin and inflection points when $x = 0$ and when x is somewhere between -2 and -1.

Now let's try to confirm these impressions using calculus. We differentiate and get

$$f'(x) = 12x^5 + 15x^4 + 9x^2 - 4x$$

$$f''(x) = 60x^4 + 60x^3 + 18x - 4$$