The maximum height is reached when $v(t) = 0$, that is, after $15/9.8 \approx 1.53$ s. Since $s'(t) = v(t)$, we antidifferentiate again and obtain

$$s(t) = -4.9t^2 + 15t + D$$

Using the fact that $s(0) = 140$, we have $140 = 0 + D$ and so

$$s(t) = -4.9t^2 + 15t + 140$$

Figure 4 shows the position function of the ball in Example 7. The graph corroborates the conclusions we reached: The ball reaches its maximum height after 1.5 s and hits the ground after 7.1 s.

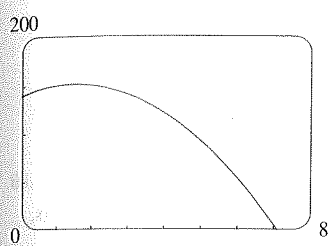


FIGURE 4

The expression for $s(t)$ is valid until the ball hits the ground. This happens when $s(t) = 0$, that is, when

$$-4.9t^2 - 15t - 140 = 0$$

Using the quadratic formula to solve this equation, we get

$$t = \frac{15 \pm \sqrt{2969}}{9.8}$$

We reject the solution with the minus sign since it gives a negative value for t. Therefore the ball hits the ground after

$$\frac{15 + \sqrt{2969}}{9.8} \approx 7.1 \text{ s}$$

3.9 Exercises

1–18 Find the most general antiderivative of the function. (Check your answer by differentiation.)

1. $f(x) = x - 3$

2. $f(x) = \frac{1}{2}x^2 - 2x + 6$

3. $f(x) = \frac{1}{2} + \frac{3}{4}x^2 - \frac{4}{5}x^3$

4. $f(x) = 8x^9 - 3x^6 + 12x^3$

5. $f(x) = (x + 1)(2x - 1)$

6. $f(x) = x(2 - x)^2$

7. $f(x) = 7x^{2/5} + 8x^{-4/5}$

8. $f(x) = x^{3.4} - 2x^{\sqrt{2}-1}$

9. $f(x) = \sqrt{2}$

10. $f(x) = \pi^2$

11. $f(x) = \dfrac{10}{x^9}$

12. $g(x) = \dfrac{5 - 4x^3 + 2x^6}{x^6}$

13. $g(t) = \dfrac{1 + t + t^2}{\sqrt{t}}$

14. $f(t) = 3\cos t - 4\sin t$

15. $h(\theta) = 2\sin\theta - \sec^2\theta$

16. $f(\theta) = 6\theta^2 - 7\sec^2\theta$

17. $f(t) = 2\sec t\,\tan t + \frac{1}{2}t^{-1/2}$

18. $f(x) = 2\sqrt{x} + 6\cos x$

19–20 Find the antiderivative F of f that satisfies the given condition. Check your answer by comparing the graphs of f and F.

19. $f(x) = 5x^4 - 2x^5, \quad F(0) = 4$

20. $f(x) = x + 2\sin x, \quad F(0) = -6$

21–40 Find f.

21. $f''(x) = 20x^3 - 12x^2 + 6x$

22. $f''(x) = x^6 - 4x^4 + x + 1$

23. $f''(x) = \frac{2}{3}x^{2/3}$

24. $f''(x) = 6x + \sin x$

25. $f'''(t) = \cos t$

26. $f'''(t) = t - \sqrt{t}$

27. $f'(x) = 1 + 3\sqrt{x}, \quad f(4) = 25$

28. $f'(x) = 5x^4 - 3x^2 + 4, \quad f(-1) = 2$

29. $f'(x) = \sqrt{x}\,(6 + 5x), \quad f(1) = 10$

30. $f'(t) = t + 1/t^3, \quad t > 0, \quad f(1) = 6$

31. $f'(t) = 2\cos t + \sec^2 t, \quad -\pi/2 < t < \pi/2, \quad f(\pi/3) = 4$

32. $f'(x) = x^{-1/3}, \quad f(1) = 1, \quad f(-1) = -1$

33. $f''(x) = -2 + 12x - 12x^2, \quad f(0) = 4, \quad f'(0) = 12$

34. $f''(x) = 8x^3 + 5, \quad f(1) = 0, \quad f'(1) = 8$

35. $f''(\theta) = \sin\theta + \cos\theta, \quad f(0) = 3, \quad f'(0) = 4$

36. $f''(t) = 3/\sqrt{t}, \quad f(4) = 20, \quad f'(4) = 7$

37. $f''(x) = 4 + 6x + 24x^2, \quad f(0) = 3, \quad f(1) = 10$

38. $f''(x) = 20x^3 + 12x^2 + 4, \quad f(0) = 8, \quad f(1) = 5$

39. $f''(x) = 2 + \cos x, \quad f(0) = -1, \quad f(\pi/2) = 0$

40. $f'''(x) = \cos x, \quad f(0) = 1, \quad f'(0) = 2, \quad f''(0) = 3$

41. Given that the graph of f passes through the point $(1, 6)$ and that the slope of its tangent line at $(x, f(x))$ is $2x + 1$, find $f(2)$.

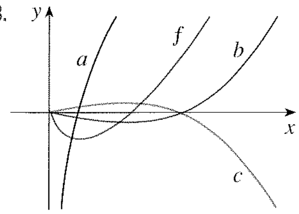
42. Find a function f such that $f'(x) = x^3$ and the line $x + y = 0$ is tangent to the graph of f.

Graphing calculator or computer required

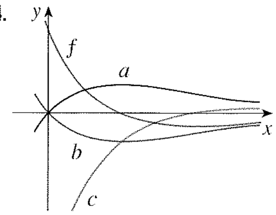
1. Homework Hints available at stewartcalculus.com

43–44 The graph of a function f is shown. Which graph is an antiderivative of f and why?

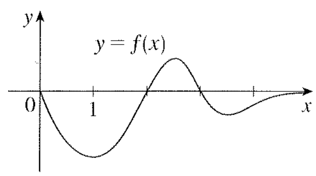
43.



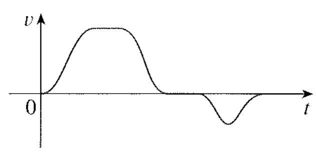
44.



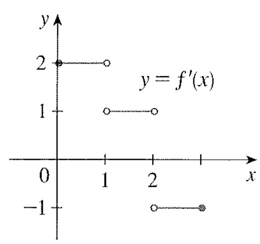
45. The graph of a function is shown in the figure. Make a rough sketch of an antiderivative F, given that $F(0) = 1$.



46. The graph of the velocity function of a particle is shown in the figure. Sketch the graph of a position function.



47. The graph of f' is shown in the figure. Sketch the graph of f if f is continuous and $f(0) = -1$.



48. (a) Use a graphing device to graph $f(x) = 2x - 3\sqrt{x}$.
(b) Starting with the graph in part (a), sketch a rough graph of the antiderivative F that satisfies $F(0) = 1$.
(c) Use the rules of this section to find an expression for $F(x)$.
(d) Graph F using the expression in part (c). Compare with your sketch in part (b).

49–50 Draw a graph of f and use it to make a rough sketch of the antiderivative that passes through the origin.

49. $f(x) = \dfrac{\sin x}{1 + x^2}, \quad -2\pi \leqslant x \leqslant 2\pi$

50. $f(x) = \sqrt{x^4 - 2x^2 + 2} - 2, \quad -3 \leqslant x \leqslant 3$

51–56 A particle is moving with the given data. Find the position of the particle.

51. $v(t) = \sin t - \cos t, \quad s(0) = 0$

52. $v(t) = 1.5\sqrt{t}, \quad s(4) = 10$

53. $a(t) = 2t + 1, \quad s(0) = 3, \quad v(0) = -2$

54. $a(t) = 3\cos t - 2\sin t, \quad s(0) = 0, \quad v(0) = 4$

55. $a(t) = 10\sin t + 3\cos t, \quad s(0) = 0, \quad s(2\pi) = 12$

56. $a(t) = t^2 - 4t + 6, \quad s(0) = 0, \quad s(1) = 20$

57. A stone is dropped from the upper observation deck (the Space Deck) of the CN Tower, 450 m above the ground.
(a) Find the distance of the stone above ground level at time t.
(b) How long does it take the stone to reach the ground?
(c) With what velocity does it strike the ground?
(d) If the stone is thrown downward with a speed of 5 m/s, how long does it take to reach the ground?

58. Show that for motion in a straight line with constant acceleration a, initial velocity v_0, and initial displacement s_0, the displacement after time t is

$$s = \tfrac{1}{2}at^2 + v_0 t + s_0$$

59. An object is projected upward with initial velocity v_0 meters per second from a point s_0 meters above the ground. Show that

$$[v(t)]^2 = v_0^2 - 19.6[s(t) - s_0]$$

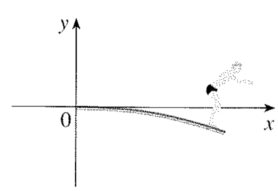
60. Two balls are thrown upward from the edge of the cliff in Example 7. The first is thrown with a speed of 15 m/s and the other is thrown a second later with a speed of 8 m/s. Do the balls ever pass each other?

61. A stone was dropped off a cliff and hit the ground with a speed of 40 m/s. What is the height of the cliff?

62. If a diver of mass m stands at the end of a diving board with length L and linear density ρ, then the board takes on the shape of a curve $y = f(x)$, where

$$EIy'' = mg(L - x) + \tfrac{1}{2}\rho g(L - x)^2$$

E and I are positive constants that depend on the material of the board and g (< 0) is the acceleration due to gravity.
(a) Find an expression for the shape of the curve.
(b) Use $f(L)$ to estimate the distance below the horizontal at the end of the board.



63. A company estimates that the marginal cost (in dollars per item) of producing x items is $1.92 - 0.002x$. If the cost of producing one item is \$562, find the cost of producing 100 items.

64. The linear density of a rod of length 1 m is given by $\rho(x) = 1/\sqrt{x}$, in grams per centimeter, where x is measured in centimeters from one end of the rod. Find the mass of the rod.

65. Since raindrops grow as they fall, their surface area increases and therefore the resistance to their falling increases. A raindrop has an initial downward velocity of 10 m/s and its downward acceleration is

$$a = \begin{cases} 9 - 0.9t & \text{if } 0 \leqslant t \leqslant 10 \\ 0 & \text{if } t > 10 \end{cases}$$

If the raindrop is initially 500 m above the ground, how long does it take to fall?

66. A car is traveling at 80 km/h when the brakes are fully applied, producing a constant deceleration of 7 m/s^2. What is the distance traveled before the car comes to a stop?

67. What constant acceleration is required to increase the speed of a car from 50 km/h to 80 km/h in 5 s?

68. A car braked with a constant deceleration of 5 m/s^2, producing skid marks measuring 60 m before coming to a stop. How fast was the car traveling when the brakes were first applied?

69. A car is traveling at 100 km/h when the driver sees an accident 80 m ahead and slams on the brakes. What constant deceleration is required to stop the car in time to avoid a pileup?

70. A model rocket is fired vertically upward from rest. Its acceleration for the first three seconds is $a(t) = 18t$, at which time the fuel is exhausted and it becomes a freely "falling" body. Fourteen seconds later, the rocket's parachute opens, and the (downward) velocity slows linearly to -5.5 m/s in five seconds. The rocket then "floats" to the ground at that rate.
(a) Determine the position function s and the velocity function v (for all times t). Sketch the graphs of s and v.
(b) At what time does the rocket reach its maximum height, and what is that height?
(c) At what time does the rocket land?

71. A high-speed bullet train accelerates and decelerates at the rate of 1.2 m/s^2. Its maximum cruising speed is 145 km/h.
(a) What is the maximum distance the train can travel if it accelerates from rest until it reaches its cruising speed and then runs at that speed for 15 minutes?
(b) Suppose that the train starts from rest and must come to a complete stop in 15 minutes. What is the maximum distance it can travel under these conditions?
(c) Find the minimum time that the train takes to travel between two consecutive stations that are 72 km apart.
(d) The trip from one station to the next takes 37.5 minutes. How far apart are the stations?

3 Review

Concept Check

1. Explain the difference between an absolute maximum and a local maximum. Illustrate with a sketch.

2. (a) What does the Extreme Value Theorem say?
(b) Explain how the Closed Interval Method works.

3. (a) State Fermat's Theorem.
(b) Define a critical number of f.

4. (a) State Rolle's Theorem.
(b) State the Mean Value Theorem and give a geometric interpretation.

5. (a) State the Increasing/Decreasing Test.
(b) What does it mean to say that f is concave upward on an interval I?
(c) State the Concavity Test.
(d) What are inflection points? How do you find them?

6. (a) State the First Derivative Test.
(b) State the Second Derivative Test.
(c) What are the relative advantages and disadvantages of these tests?

7. Explain the meaning of each of the following statements.
(a) $\lim_{x\to\infty} f(x) = L$ (b) $\lim_{x\to-\infty} f(x) = L$ (c) $\lim_{x\to\infty} f(x) = \infty$
(d) The curve $y = f(x)$ has the horizontal asymptote $y = L$.

8. If you have a graphing calculator or computer, why do you need calculus to graph a function?

9. (a) Given an initial approximation x_1 to a root of the equation $f(x) = 0$, explain geometrically, with a diagram, how the second approximation x_2 in Newton's method is obtained.
(b) Write an expression for x_2 in terms of x_1, $f(x_1)$, and $f'(x_1)$.
(c) Write an expression for x_{n+1} in terms of x_n, $f(x_n)$, and $f'(x_n)$.
(d) Under what circumstances is Newton's method likely to fail or to work very slowly?

10. (a) What is an antiderivative of a function f?
(b) Suppose F_1 and F_2 are both antiderivatives of f on an interval I. How are F_1 and F_2 related?