METU - NCC

	Precalculus Final	
Code : *Math 100* Acad.Year: *2012-2013* Semester : *Spring* Date : *03.06.2013* Time : *16:00* Duration : *120 minutes*	Last Name: Name : Department: Signature:	Student No.: Section:
	10 QUESTIONS ON 4 PAGES TOTAL 100 POINTS	

1 (6)	2 (9)	3 (12)	4 (8)	5 (8)	6 (10)	7 (10)	8 (12)	9 (8)	10 (9)	11 (8)	

1. *(6 pts)* Find the equation of the line passing through the points (2,-2) and (-1,4).

$m = \frac{4-(-2)}{-1-(2)} = \frac{6}{-3} = -\frac{6}{3} = -2$

$y-4 = -2(x-(-1))$

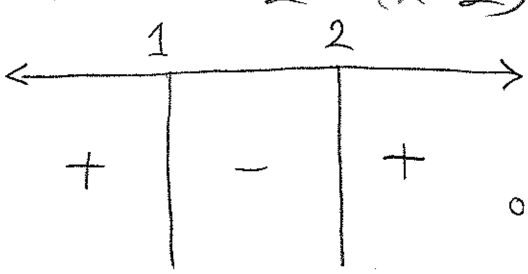
$y = -2x+2$

$\boxed{y=-2x+2}$

2. *(9 pts)* Find the domain of $f(x) = \frac{4}{\sqrt{x^2-3x+2}}$.

To define f at x, we need $x^2-3x+2>0$

$x^2-3x+2 = (x-2)\cdot(x-1) = 0 \Rightarrow x=2,\ x=1$



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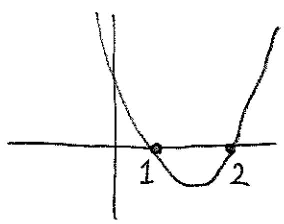


$\boxed{(-\infty, +1) \cup (2, +\infty)}$

3. *(12 pts)* Solve each equation.

(a) $2^{2x} - 2^x = 2$ $\Rightarrow (2^x)^2 - 2^x - 2 = 0$ Let $u = 2^x$;

$u^2 - u - 2 = 0 \Rightarrow (u-2)\cdot(u+1) = 0 \Rightarrow u=2$, $u=-1$

$2^x = 2 \Rightarrow x = 1$

$2^x = -1$ doesn't have any solution

$\boxed{x=1}$

(b) $\ln(x+2) + \ln(x-1) = \ln(3x+1)$

$\ln((x+2)\cdot(x-1)) = \ln(3x+1)$ since lnx is one to one $\Rightarrow (x+2)(x-1) = 3x+1$

So, $x^2+x-2 = 3x+1$, $x^2-2x-3=0 \Rightarrow (x-3)\cdot(x+1)=0$

$x=3$ or $x=-1$

Check: $x=3$ ✓ $\ln(5)+\ln(2)=\ln(10)$

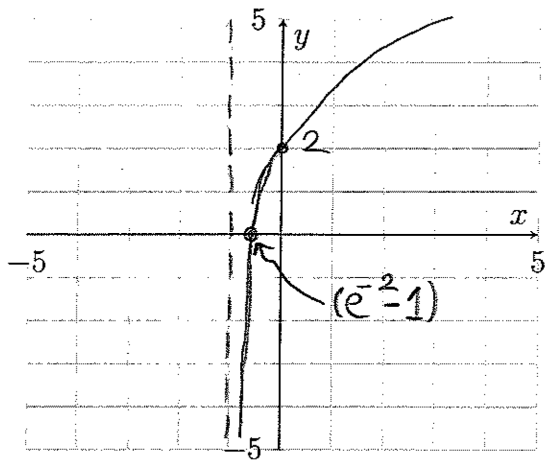
$x=-1$ X since $\ln(-1-1)$ is not defined!

$\boxed{x=3}$

4. *(8 pts)* Sketch the graph of $f(x) = 2 + \ln(x+1)$ on the follwing plot.

$\ln(x) \xrightarrow[\text{1 unit}]{\text{shift left}} \ln(x+1)$

$\ln(x+1) \xrightarrow[\text{2 units}]{\text{shift up}} 2 + \ln(x+1)$



$\ln(x+1) = -2$

$x+1 = e^{-2}$

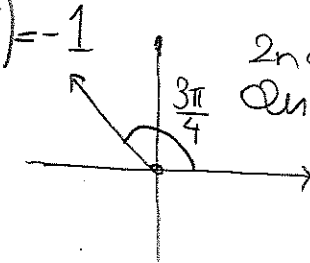
$x = e^{-2} - 1$

5. *(9 pts)* Find the following values.

(a) $\sin(\frac{91\pi}{4}) = \sin\left(\frac{3\pi}{4}\right) = \frac{\sqrt{2}}{2}$ $\qquad$ $\cot(\frac{91\pi}{4}) = \cot\left(\frac{3\pi}{4}\right) = -1$

$$\begin{array}{r|l} 91 & 8 \\ \hline -88 & 11 \\ \hline 3 & \end{array}$$

$\frac{91\pi}{4} = \frac{88\pi}{4} + \frac{3\pi}{4} = 11\cdot(2\pi) + \frac{3\pi}{4}$



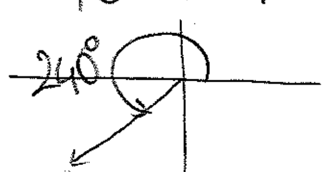
2nd Quadrant

(b) $\tan(2760°) = \tan(240°) = \sqrt{3}$ $\qquad$ $\cos(2760°) = \cos(240°) = -\frac{1}{2}$

$$\begin{array}{r|l} 2760 & 360 \\ \hline -2520 & 7 \\ \hline 240 & \end{array}$$

2100
420

$2760° = 7 \cdot 360° + 240° \Rightarrow$ reference angle is $60°$



3^{rd} Quadrant

6. *(10 pts)* Draw the graph of $y = -2\sin(\pi x - \frac{\pi}{2})$ on the following plot.

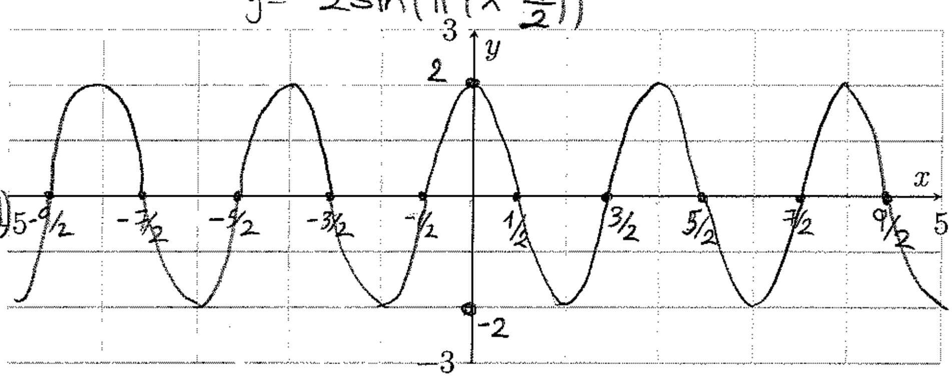
$y = -2\sin\left(\pi\left(x - \frac{1}{2}\right)\right)$

$A = -2$

$|A| = 2$ Amplitude

$T = \frac{2\pi}{\pi} = 2$ (Period)

$\frac{1}{2}$ = Phase Shift



7. *(10 pts)* Compute the following values by using related trigonometric formulas. Show your work.

(a) $\sin(105°) = \sin(60°+45°) = \sin(60°)\cdot\cos(45°) + \sin(45°)\cdot\cos(60°)$

$\boxed{\sin(a+b) = \sin a\cdot\cos b + \sin b\cdot\cos a}$

$$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$$

$$= \frac{\sqrt{6}+\sqrt{2}}{4}$$

$$\boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$$

(b) $\cos(52.5°) = \cos\left(\frac{105°}{2}\right) = \sqrt{\frac{1}{2} + \cos(105°)} = \sqrt{\frac{1}{2} - \sqrt{1-\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)^2}}$

$\boxed{\cos(2a) = 2\cos^2 a - 1 \ \& \ \cos^2(2a)+\sin^2(2a) = 1}$

$$\cos(105°) = -\sqrt{1-\sin^2(105°)}$$

$$= -\sqrt{1-\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)^2}$$

$$\boxed{\sqrt{\frac{1}{2} - \sqrt{1-\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)^2}}}$$

(must be negative since 105° is in 2nd quadrant)

8. *(12 pts)* Find all x which satisfy the following equation.

$$\tan^2(x) + \sec(x) = 1$$

$\tan^2 x = \sec^2 x - 1 \Rightarrow$ $\sec^2 x - 1 + \sec x = 1$

$\sec^2 x + \sec x - 2 = 0$

let $u = \sec x$ $\quad u^2 + u - 2 = 0$

$(u+2)(u-1) = 0$

$u = -2$ or $u = 1$

$\sec x = -2 \Rightarrow \frac{1}{\cos x} = -2 \Rightarrow \cos x = -\frac{1}{2} \Rightarrow x = \frac{2\pi}{3} + 2\pi k$

or

$x = \frac{4\pi}{3} + 2\pi k$

$\sec x = 1 \Rightarrow \cos x = 1 \quad x = 0 + 2\pi k$

$\frac{2\pi}{3} + 2\pi k$

OR $\frac{4\pi}{3} + 2\pi k$

OR $0 + 2\pi k$

9. *(8 pts)* Compute the following values.

(a) $\sin(\arctan(1)) =$ $\sin(\pi/4) = \frac{\sqrt{2}}{2}$

(b) $\tan^{-1}(\tan(\frac{3\pi}{4})) =$ $\tan^{-1}(-1) = -\frac{\pi}{4}$

10. *(9 pts)* Use basic trigonometric identities to verify the following identity.

$$\frac{\cos x}{(1+\sin x)} + \tan x = \sec x$$

$$LHS = \frac{\cos x}{1+\sin x}\,(\cos x) + \frac{\sin x}{\cos x}\,(1+\sin x) = \frac{\cos^2 x + (\sin^2 x + \sin x)}{(1+\sin x)\cdot\cos x}$$

$$= \frac{\cos^2 x + \sin^2 x + \sin x}{(1+\sin x)\cdot\cos x}$$

$(\sin^2 x + \cos^2 x = 1)$

$$= \frac{\cancel{(1+\sin x)}}{\cancel{(1+\sin x)}\cdot\cos x}$$

$$= \frac{1}{\cos x}$$

$$= \sec x \quad \checkmark$$

11. *(8 pts)* Determine whether the following statements or equations are TRUE or FALSE. **Circle** your answer.

T F (a) A periodic function can't be one to one.

T F (b) If $f(x)$ is one to one, then $f^{-1}(x)$ may not be one to one.

T F (c) Domain of $f \circ g$ is equal to Dom$(f)\cap$Dom(g).

T F (d) For all x, $|\arctan(x)| < \frac{\pi}{2}$.

T F (e) If the side lengths of a triange are 2,3,4, then one angle must be obtuse.

T F (f) The period of $\arcsin(x)$ is equal to 2π.

T F (g) The point $(4, 1)$ is on the circle with center $(1, 2)$ and radius 5.

T F (h) The sum of the absolute values of two numbers is the same as the absolute value of the sum of the two numbers.