M E T U – N C C
Mathematics Group

	Calculus with Analytic Geometry **Final Exam**	

Code : MAT 119 Acad.Year : 2010-2011 Semester : Spring Instructors : A.D./H.T./B.W. Date : 09.06.2011 Time : 13.00 Duration : 120 minutes	Last Name : Name : Stud. No : Dept. : Sec. No : Signature : **6** Questions on **8** Pages Total **100** Points

1 (18)	2 (24)	3 (12)	4 (15)	5 (21)	6 (10)		

Q.1 (6 × 3 = 18 pts) Evaluate the following limits:

(a) $\lim\limits_{x\to 0} \dfrac{\int_x^0 (\mathrm{e}^t + t - 1)\,\mathrm{d}t}{x^2}$

(b) $\lim\limits_{x\to\infty} (1 + \mathrm{e}^{-x})^x$

(c) $\lim_{x \to 0} \dfrac{\cos x^2}{x^2}$

(d) $\lim_{x \to 0} \dfrac{\sin x^2 - x^2}{x^6}$

(e) $\lim_{x \to \infty} (\ln x)^{1/x}$

(f) $\lim_{x \to 0^+} x^{\csc x}$

Last Name: **Name:**

Q.2 (6 × 4 = 24 pts) Evaluate the following integrals:

(a) $\displaystyle\int \frac{\sqrt{x+1}}{x-3}\,\mathrm{d}x$

(b) $\displaystyle\int x^{3/2}\ln(5x)\,\mathrm{d}x$

(c) $\displaystyle\int \sin^3(2x)\cos^4(2x)\,\mathrm{d}x$

(d) $\displaystyle\int \frac{x-8}{x^3+4x}\,\mathrm{d}x$

(e) $\displaystyle\int \frac{1}{(x^2+4x+5)^{3/2}}\,\mathrm{d}x$

(f) $\displaystyle\int \frac{x^3-1}{\sqrt{1-x^2}}\,\mathrm{d}x$

Q.3 (3 × 4 = 12 pts) Determine whether the following integrals converge or diverge. If they converge, what do they converge to?

(a) $\displaystyle\int_0^4 \frac{x}{x^2-4}\,\mathrm{d}x$

(b) $\displaystyle\int_4^\infty \frac{1}{\sqrt{x}\mathrm{e}^{\sqrt{x}}}\,\mathrm{d}x$

(c) $\displaystyle\int_1^\infty \frac{\sin^4 x+1}{x^{1/4}}\,\mathrm{d}x$

Q.4 (7 + 8 = 15 pts) Consider the region between the curves $y = x^2$ and $y = 2 - x^2$ from $x = 0$ to $x = 2$.

(a) Find the volume of the solid obtained by rotating this region about the y-axis.

(b) Find the volume of the solid obtained by rotating this region about the line $y = 4$.

Q.5 (3 × 7 = 21 pts) The following parts involve the curve $y = x \tan x$ from $y = 0$ to $y = \pi/4$.

(a) Write, but **do NOT** evaluate, the integral which gives the **arclength** of $y = x \tan x$ from $y = 0$ to $y = \pi/4$.

(b) Write, but **do NOT** evaluate, the integral which gives the **surface area** of the surface obtained by rotating $y = x \tan x$ from $y = 0$ to $y = \pi/4$. about the y-axis.

(c) Write, but **do NOT** evaluate, the integral which gives the **surface area** of the surface obtained by rotating $y = x \tan x$ from $y = 0$ to $y = \pi/4$. about the line $y = -1$.

Q.6 (10 pts) Use the mean value theorem (MVT) to show that $\ln x < x - 1$ for $x > 1$.

NOTE. This problem was included also in the 2nd Midterm Exam, which is supposed to be somewhat comrehensive.