	Basic Linear Algebra II. Midterm	

Code : *Math 260* Acad.Year: *2010-2011* Semester : *Spring* Date : *7.5.2011* Time : *10:00* Duration : *120 minutes*	Last Name: Name : Student No: Department: Section: Signature: 6 QUESTIONS ON 6 PAGES TOTAL 100 POINTS

1. (14)	2. (16)	3. (16)	4. (24)	5. (21)	6. (9)		

Show your work! No calculators! Please draw a box around your answers!
Please do not write on your desk!

1. (14 pts) The following parts are about linear transformations.

(a) (4 pts) Let T be $90°$ rotation around the x-axis in $\mathbb{R}^3$ (taking the positive y-axis to the positive z-axis). Write the matrix $A = [T]$ of the transformation T.

(b) (4 pts) Write the matrix B which reflects $\mathbb{R}^3$ across the xy-plane.

(c) (2 pts) Compute $T_{BA}(x, y, z)$ (T_{BA} is the the transformation whose matrix is BA).

$$T_{BA}(x, y, z) = \left(\boxed{}, \boxed{}, \boxed{} \right)$$

(d) (4 pts) Geometrically describe the action of the linear transformation T_{BA}.
(i.e. Is it a reflection (across what?), or rotation (how much?), or scaling, etc.?)

2. (16 pts) The following parts all deal with the vectors $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ where $\mathbf{v}_1 = (1, -1, 0)$, $\mathbf{v}_2 = (1, 0, 2)$, and $\mathbf{v}_3 = (0, 0, 1)$.

(a) (4 pts) Show that $\mathcal{B}$ is a basis for $\mathbb{R}^3$.

(b) (4 pts) Find the coordinates of $(4, 2, 5)$ relative to the basis $\mathcal{B}$.

(c) (4 pts) Consider the inner product on $\mathbb{R}^3$ coming from $\mathcal{B}$ by setting $\langle \mathbf{v}_i, \mathbf{v}_j \rangle = 0$ for all $i \neq j$ and $\langle \mathbf{v}_i, \mathbf{v}_i \rangle = 1$ for each i.
What is the norm of $(4, 2, 5)$ with respect to this inner product?

(d) (4 pts) Compute $\big\langle (4, 2, 5),\ (1, 0, 0) \big\rangle$ using the inner product from (c).

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3. **(16 pts)** For each part below, decide which of the following is true:

  I. V is not a vector space.
  II. V is a vector space but W is not a subspace.
  III. V is a vector space and W is a subspace.

If V is not a vector space, then indicate why not. Either show W is a subspace, or give an example why it isn't.

(a) (4 pts) $V = \mathbb{R}^3$ with the usual operations. $W = \{(a, b, c) \mid \ a - 2b + c = 0\}$.

(b) (4 pts) $V = \mathbb{Z}$ with the usual operations. W is the set of all even integers.

(c) (4 pts) V is continuous functions on $\mathbb{R}$ with the usual operations. W is the subset of positive functions (i.e. $f(x) \geq 0$).

(d) (4 pts) $V = \mathbb{R}^2_+$ (i.e. $V = \{(x, y) \mid \ x > 0,\ y > 0\}$) with the operations $(x, y) \oplus (z, w) = (xz, yw)$ and $k \otimes (x, y) = (x^k, y^k)$. W is the set of points (x, y) on the parabola $y = x^2$ in V.

4. (24 pts) Give a short (one sentence) answer for each of the following parts.

(a) None of the following sets of vectors span $\mathcal{P}_2$ (the vector space of degree 2 polynomials). For each, give a simple reason why not.

(i) (3 pts) $\{1 + x^2,\ 1 + x\}$

(ii) (3 pts) $\{0,\ 1 + x,\ 1 + x + x^2\}$

(iii) (3 pts) $\{1 + x,\ x + x^2,\ 2 + 2x,\ 3x + 3x^2\}$

(iv) (3 pts) $\{x,\ x + x^2,\ 2x + 3x^2,\ -x + 2x^2\}$

(b) None of the following sets of symmetric 2×2 matrices are linearly independent. For each, give a simple reason why not.

(i) (3 pts) $\left\{ \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 1 \\ 1 & 3 \end{bmatrix}, \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \right\}$

(ii) (3 pts) $\left\{ \begin{bmatrix} 1 & 2 \\ 2 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} -1 & 3 \\ 3 & 0 \end{bmatrix} \right\}$

(iii) (3 pts) $\left\{ \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}, \begin{bmatrix} 2 & 4 \\ 4 & 6 \end{bmatrix} \right\}$

(iv) (3 pts) $\left\{ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \right\}$

5. (21 pts) The following parts deal with the matrix A which reduces as given below:

$$A = \begin{bmatrix} 2 & 3 & -1 & 1 & -1 & 1 & 2 \\ 1 & 0 & 1 & 2 & 4 & 2 & 4 \\ 3 & 3 & 0 & 3 & 3 & 3 & 6 \\ 0 & -1 & 1 & 1 & 3 & 3 & 8 \\ -1 & 3 & -4 & 3 & -5 & 4 & 1 \end{bmatrix}$$

$$\text{rref}(A) = \begin{bmatrix} 1 & 0 & 1 & 0 & 2 & 0 & 2 \\ 0 & 1 & -1 & 0 & -2 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 & 0 & -2 \\ 0 & 0 & 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) (5 pts) Give a basis for the column space of A.

(b) (2 pts) Write a nontrivial (some constants nonzero) relation among the column vectors of A:

$$0 = \square(\text{Col1}) + \square(\text{Col2}) + \square(\text{Col3}) + \square(\text{Col4}) + \square(\text{Col5}) + \square(\text{Col6}) + \square(\text{Col7})$$

(c) (5 pts) Give a basis for the row space of A.

(d) (5 pts) Give a basis for the null space of A.

(e) (2 pts) What is the rank of A?

(f) (2 pts) Do the columns of A span $\mathbb{R}^5$? Explain.

6. (9 pts) Consider an electrical circuit formed by a 2×2 grid. The standard way to write the KVL equations for this circuit is to write them for the four small loops around grid squares, as in figure 1. Do we get the same result if we replace these equations by the following four: the two 1×2 loops and the two 2×1 loops, as in figure 2? Explain.

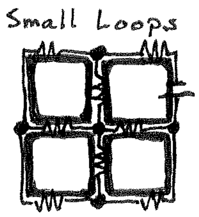

Figure 1.

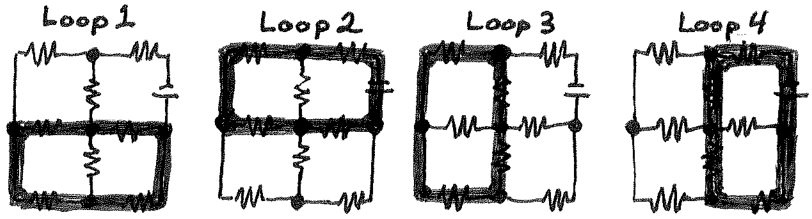

Figure 2.