METU NCC
MAT 260 Mid-Term II
Duration: 120 minutes

Last Name:..**First Name**...

Student number:..

Instructions:

1. Check that you have all pages, which are numbered sequentially (6 pages in total)
2. Marks are shown in brackets
3. Show all significant steps. Few marks (if any) will be given for answers alone.

Question	Mark
1	
2	
3	
4	
5	
Total	

[8 + 8 + 4 points] **1.** Let

$$A = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 1 & 0 & 1 & 4 \\ 1 & 0 & -1 & 2 \\ 1 & 1 & 2 & 7 \end{bmatrix}$$

(a) Find a basis for the nullspace of A

(b) Find a basis for the row space of A

(c) What is the rank of A and the nullity of A

$\left[10+10 \text{ points}\right]$ **2.** Let $W = \{(a,b,c,d) \in \mathbf{R}^4 : a+b+c = d\}$

(a) Show that W is a subspace of $\mathbf{R}^4$.

(b) Find a basis for the vector space W, and find dim (W).

[10 + 10 points] 3. Let $v = (2,-1,3)$, $v_1 = (1,0,0)$, $v_2 = (2,2,0)$, $v_3 = (3,3,3)$

(a) Show that the set $S = \{v_1, v_2, v_3\}$ is a basis for $\mathbf{R}^3$.

(b) Find the coordinate vector of v relative to the basis S.

$\left[15+5 \text{ points}\right]$ **4.** (a) Find the standard matrix for the linear transformation $T : \mathbf{R}^2 \to \mathbf{R}^2$ that first rotates a vector through an angle $\alpha = \frac{\pi}{3}$, followed by an orthogonal projection on the x –axis, followed by a reflection about the line $y = x$

(b) Find the standard matrix for T^{-1}

[20 points] **5.** For each of the following statements write **T** or **F** inside the parenthesis if the

statement is true or false respectively.

(1) (   ) T is a transformation from $\mathbf{R}^n$ to $\mathbf{R}^m$, and $T(0) = 0$, then T is linear.

(2) (   ) If $\{ v_1, v_2 \}$ is a linearly dependent set, then $\{ v_1 + 2v_2, 3v_1 - v_2 \}$ is also linearly dependent.

(3) (   ) If $\dim(V) = n$, then n is the largest number of linearly dependent vectors in V

(4) (   ) If $\|u+v\|^2 = \|u\|^2 + \|v\|^2$, then u and v are orthogonal

(5) (   ) The transformation $T(x, y) = (x_1 + x_2, x_1 + x_1 x_2)$ is linear

(6) (   ) The plane $x - y = 0$ in $\mathbf{R}^3$ has basis vectors $(1,1,0)$ and $(0,0,1)$

(7) (   ) If A is $n \times n$ matrix with $\det(A) \neq 0$, then the nullspace of A does not contain any non-zero vectors.

(8) (   ) Five non-zero vectors in $\mathbf{R}^4$ must be linearly dependent.

(9) (   ) The set of vectors $S = \{(-1,18,7),(-1,4,1),(1,3,2)\}$ is a basis for $\mathbf{R}^3$

(10) (   ) If Span(S_1) =Span(S_2) then $S_1 = S_2$.