METU NCC
MAT 260 Mid-Term I
Duration: 100 minutes

Last Name:..**First Name**...

Student number:...

Instructions:

1. Check that you have all pages, which are numbered sequentially (6 pages in total)
2. Marks are shown in brackets
3. Show all significant steps. Few marks (if any) will be given for answers alone.

Question	Mark
1	
2	
3	
4	
5	
Total	

[20 points] 1. Let

$$A = \begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 \\ 2 & 6 & -5 & -2 & 4 & -3 \\ 0 & 0 & 5 & 10 & 0 & 15 \\ 2 & 6 & 0 & 8 & 4 & 18 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 0 \\ -1 \\ 5 \\ 6 \end{bmatrix}$$

Use the <u>Gauss-Jordan Elemination Method</u> to solve the linear system $AX = b$.

[20 points] 2. Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$$

Determine whether A is invertible and if so find the inverse matrix A^{-1} using

elementary row operations.

[20 points] 3. Evaluate the following determinants by using <u>row reduction and properties of determinants</u>

$$(a)\quad \begin{vmatrix} 3 & -6 & 9 \\ -2 & 7 & -2 \\ 0 & 1 & 5 \end{vmatrix}$$

$$(b)\quad \begin{vmatrix} 1 & 3 & 1 & 5 & 3 \\ -2 & -7 & 0 & -4 & 2 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{vmatrix}$$

[20 points] 4. Given the matrix

$$A = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \\ d_1 & d_2 & d_3 & d_4 \end{bmatrix} \text{ and } \det(A) = 2$$

Evaluate the following determinants

(a) $\det(3A^{-1})$

(b) $\det(A^{-2})$

(c) $\det\left((A^3)^T\right)$

(d) $\begin{bmatrix} -d_1 & -d_2 & -d_3 & -d_4 \\ 2b_1 & 2b_2 & 2b_3 & 2b_4 \\ 3c_1 & 3c_2 & 3c_3 & 3c_4 \\ -a_1 & -a_2 & -a_3 & -a_4 \end{bmatrix}$

[20 points] 5. For each of the following statements write T or F inside the parenthesis if the statement is true or false respectively.

(1) ( ) A matrix with a column of zeros must have an inverse

(2) ( ) If the reduced row-echelon form of the augmented matrix for a linear system has a row of zeros, then the system has infinitely many solutions

(3) ( ) If A and B are $n \times n$ matrices, then $(A+B)^2 = A^2 + 2AB + B^2$

(4) ( ) If A is an invertible matrix and B is obtained from A by row operations, then B is also invertible

(5) ( ) Every square matrix can be expressed as a product of elementary matrices

(6) ( ) If A is $n \times n$ matrix such that $A^2 - 3A + I_n = 0$, then $A^{-1} = A - 3I$

(7) ( ) $\det(I_n + A) = 1 + \det(A)$

(8) ( ) If $\det(A) = 0$, then the homogeneous system $AX = 0$ has infinitely many solutions.

(9) ( ) If A is a square matrix and B is obtained from A by a single row operation, then we necessarily have $\det(B) = \det(A)$

(10) ( ) There is no square matrix A such that $\det(AA^T) = -1$