METU NCC
MAT 260 Final Exam
Duration: 120 minutes

Last Name:...First Name..

Student number:..

Instructions:

1. **Check that you have all pages, which are numbered sequentially (6 pages in total)**
2. **Marks are shown in brackets**
3. **Show all significant steps. Few marks (if any) will be given for answers alone.**

Question	Mark
1	
2	
3	
4	
5	
Total	

[8 + 8 + 4 points] 1. Let

$$A = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$$

(a) Find a basis for the nullspace of A

(b) Find a basis for the row space of A

(c) Find the rank of A and the nullity of A

[5 + 10 + 5 points] **2.** Let

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

(a) Find the eigenvalues of A

(b) Is A diagonalizable? If yes, find a matrix P that diagonalizes A.

(c) Compute A^{10}

[5 + 10 + 5 points] **3.** Let $B = \{v_1 = (1,1,1),\ v_2 = (-1,1,0),\ v_3 = (1,2,1)\}$

(a) Show that the set $B = \{v_1, v_2, v_3\}$ is a basis for $\mathbf{R}^3$.

(b) Transform the basis B to obtain an orthonormal basis.

(c) Express $u = (-1,0,0)$ as a linear combination of the vectors in the orthonormal basis obtained in part (b).

[10 + 10 points] 4. Let $W = span\{ w_1, w_2, w_3 \} \subseteq \mathbf{R}^4$ where

$$w_1 = \left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, 0, \frac{3}{\sqrt{14}}\right), \quad w_2 = \left(\frac{2}{7}, -\frac{4}{7}, \frac{5}{7}, \frac{2}{7}\right), \quad w_3 = \left(\frac{4}{\sqrt{21}}, \frac{1}{\sqrt{21}}, 0, -\frac{2}{\sqrt{21}}\right)$$

(*a*) Show that $S = \{ w_1, w_2, w_3 \}$ is orthonormal with respect to the Standard inner product

(*b*) Let $u = (1, -1, 0, 1)$. Find $proj_W\, u$, i.e the orthogonal projection of u onto W.

[20 points] **5.** For each of the following statements write **T** or **F** inside the parenthesis if the statement is true or false respectively.

(1) ( ) If T is a transformation from $\mathbf{R}^2$ to $\mathbf{R}^2$, then $T(x_1, x_2) = (x_1 + x_2, x_2)$ is linear.

(2) ( ) If A is 3×4 matrix, then the column vectors of A are linearly dependent.

(3) ( ) The matrix A and its transpose have the same eigenvalues.

(4) ( ) A finite set of vectors that contains the zero vector is linearly independent.

(5) ( ) $\lambda = 0$ is an eigenvalue of a matrix A if and only if A is not invertible.

(6) ( ) The set $W = \{(a, 1, b) : a, b \in \mathbf{R}\}$ is a subspace of $\mathbf{R}^3$.

(7) ( ) If $\det(A) = 0$, then the homogeneous system $AX = 0$ has infinitely many solutions.

(8) ( ) A linear system of five equations in three unknowns cannot be consistent

(9) ( ) If A is a triangular matrix, then the sum of its eigenvalues is equal to the sum of its diagonal elements.

(10) ( ) If A^2 is symmetric, then so is A.