METU NCC

Math 260 Basic Linear Algebra Exam II 07.05.2008		
Last Name: Name : Student No:	Instructure: *R.B., M.K., D.M.* Time : 17: 40 Duration : 110 *minutes*	Signature
6 QUESTIONS ON 6 PAGES		TOTAL 60 POINTS

1	2	3	4	5	6			

Question 1(10 pts.) Determine whether following sets are subspaces of $\mathbb{R}^3$.

(a) $\{ (a, b, 2a - b) \mid a, b$ are real numbers$\}$.

(b) $\{ (a, b, a + 1) \mid a, b$ are real numbers$\}$.

Question 2 (10 pts.) For each of the following statements write T or F inside the parenthesis if the statement is true or false respectively.

(1) ( ) If span(S_1) =span(S_2), then S_1 and S_2 have the same number of elements.

(2) ( ) If S is a linearly independent set, then any subset of S is also linearly independent.

(3) ( ) All basis of $\mathbb{R}^2$ have two elements.

(4) ( ) The map $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T((x, y) = (x + y + 1, x - y)$ is linear.

(5) ( ) $(a\cos\theta + b\sin\theta)^2 \leq a^2 + b^2$ for all real numbers a, b and θ.

(6) ( ) If $||v + v||^2 = ||u||^2 - ||v||^2$, then u and v are orthogonal.

(7) ( ) Let v be a vector and k be a positive integer. Then $\{v, kv\}$ is linearly dependent.

(8) ( ) The orthogonal projection onto the y-axis in $\mathbb{R}^2$ is one-to-one.

(9) ( ) The set $\{(1, a) \mid a \in \mathbb{R}\}$ is a subspace of $\mathbb{R}^2$.

(10) ( ) Let A be a 2×2 matrix and $b = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. If $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and $Y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ are two solutions of $AX = b$, then $X + Y$ is also a solution.

Question 3(10 pts.) (a) Find the rank and the nullity of the matrix $\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 1 & 1 \\ 3 & -3 & -5 \end{bmatrix}$.

(b) Find a basis for the null space of $\mathbf{B} = \begin{bmatrix} 1 & 3 & -1 & 1 \\ 2 & 6 & -3 & 0 \end{bmatrix}$.

(c) Determine whether $v = (1, 0, 0)$ is in the row space of the matrix $\mathbf{C} = \begin{bmatrix} 0 & 0 & -1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, Question 4(10 pts.) (a) Let $v_1 = (1, 0, 2, 0, 1), v_2 = (2, 0, -1, 3, 0), v_3 = (3, 0, 1, 3, 1), v_4 = (-4, 0, 2, -6, 0)$. Find a basis for the subspace of $\mathbb{R}^5$ spanned by $\{v_1, v_2, v_3, v_4\}$.

(b) Let $w_1 = (1, 0, 2), w_2 = (2 - 1, 3), w_3 = (0, 1, 2)$. Determine whether $\{w_1, w_2, w_3\}$ is a basis for $\mathbb{R}^3$.

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Question 5 (10 pts.) (a) Let $V = M_{2,2}$ be the space of 2×2 matrices. Determine whether $F : V \to V$ given by $F(A) = A^T$ is linear. Here, A^T denotes the transpose of A.

(b) Is the map $T : \mathbb{R}^3 \to \mathbb{R}$ defined by $T(x, y, z) = x + 2y + z$ linear?

Question 6 (10 pts.) Let T be the reflection about xy-plane and let P be the orthogonal projection onto the yz-plane.

(a) Find the standard matrices T, P and $P \circ T$.

(b) Determine whether T, P and $P \circ T$ are one-to-one.