METU NCC

Math 260 Basic Linear Algebra Exam I 28.03.2008		
Last Name : Name : Student No:	Instructure: *R.B., M.K., D.M.* Time : 17: 40 Duration : 100 *minutes*	Signature
6 QUESTIONS ON 6 PAGES		TOTAL 60 POINTS

1	2	3	4	5	6			

Question 1(10 pts.) Let $\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 3 & -2 \\ 2 & 4 & 1 & 7 & 0 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 4 \\ 3 \\ 7 \\ 7 \end{bmatrix}$. Solve the linear system $\mathbf{Ax} = \mathbf{b}$.

Question 2 (10 pts.) For each of the following statements write T or F inside the parenthesis if the statement is true or false respectively.

(1) ( ) A linear system of five equations in three unknowns cannot have any solution.

(2) ( ) If the reduced row-echelon form of the augmented matrix for a linear system has a row consisting of zeroes, then the system must have infinitely many solutions.

(3) ( ) For every square matrix A, the linear system $Ax = 0$ has a unique solution $x = 0$.

(4) ( ) If A and B are two $n \times n$ matrices, then $(A + B)^2 = A^2 + 2AB + B^2$.

(5) ( ) If A^{-1} exists and $AB = 0$, then $B = 0$.

(6) ( ) If A has a row of zeroes and if AB is defined. then AB has a row consisting of zeroes.

(7) ( ) If A and B are invertible matrices, then $A + B$ is also invertible.

(8) ( ) If A is an $n \times n$ matrix suct that $A^2 - 3A + I = 0$, then $A^{-1} = A - 3I$.

(9) ( ) If A is an invertible matrix and if B is obtained from A by row operations, then B is also invertible.

(10) ( ) If A is a square matrix and if B is obtained from A by a single row operation, then we necessarily have $\det(B) = \det(A)$.

Question 3 (10 pts.) Find $\mathbf{A}^{-1}$ if $\mathbf{A} = \begin{bmatrix} 3 & -4 & 5 \\ 2 & -3 & 1 \\ 3 & -5 & -1 \end{bmatrix}$.

Question 4.(10 pts.) Evaluate the following determinants.

(a) $$\begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix}$$

(b) $$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{vmatrix}$$

Question 5.(10 pts.) Given the matrix $\mathbf{A} = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \\ d_1 & d_2 & d_3 & d_4 \end{bmatrix}$ and $\det(\mathbf{A}) = 4$, evaluate the following determinants.

$\det(-3\mathbf{A}^T)$

$\det(\mathbf{A}^{-1})$

$\det(\mathbf{A}^4)$

$\det(\mathbf{A}^{-5})$

$$\begin{vmatrix} -a_1 & -a_2 & -a_3 & -a_4 \\ b_1 & b_2 & b_3 & b_4 \\ 3c_1 & 3c_2 & 3c_3 & 3c_4 \\ 2d_1 & 2d_2 & 2d_3 & 2d_4 \end{vmatrix}$$

$$\begin{vmatrix} c_1 & c_2 & c_3 & c_4 \\ b_1 & b_2 & b_3 & b_4 \\ a_1 & a_2 & a_3 & a_4 \\ d_1 & d_2 & d_3 & d_4 \end{vmatrix}$$

$$\begin{vmatrix} a_1 - 7a_3 & a_2 & a_3 & a_4 \\ 5(b_1 - 7b_3) & 5b_2 & 5b_3 & 5b_4 \\ c_1 - 7c_3 & c_2 & c_3 & c_4 \\ d_1 - 7d_3 & d_2 & d_3 & d_4 \end{vmatrix}$$

Question 6.(10 pts.) Determine whether each of the following linear systems has no solutions, a unique solution or infinitely many solutions, explaining your answer.

(a)

$2x + y + z = 1$

$x + y - z = 2$

$x + y + 2z = 0.$

(b)

$2x + y + z = 0$

$x + y - z = 0$

$x - y + 5z = 0.$

(c)

$2x + y + z = 1$

$x + y - z = 2$

$x - y + 5z = 4.$