METU NCC FINAL

Math 260 Basic Linear Algebra ~~Exam II~~ 09.06.2008		
Last Name: Name : Student No:	Instructure: *R.B., M.K., D.M.* Time : 16:30 Duration : 120 *minutes*	Signature

6 QUESTIONS ON 6 PAGES						TOTAL 70 + 4 POINTS		
1	2	3	4	5	6	Show the details of your work		

Question 1 (15 pts.) For each of the following statements write T or F inside the parenthesis if the statement is true or false respectively.

(1) ( ) A linear system of five equations in three unknowns cannot have any solution.

(2) ( ) Every homogeneous linear system has at least one solution.

(3) ( ) For every square matrix A, the linear system $Ax = 0$ has a unique solution $x = 0$.

(4) ( ) If A and B are two $n \times n$ matrices, then $(AB)^2 = A^2B^2$.

(5) ( ) Let A and B be 2×2 matrices. If $AB = 0$, then either $\det(A) = 0$ or $\det(B) = 0$.

(6) ( ) There is no matrix A of size 3×6 whose rank is 5.

(7) ( ) If A is a square matrix and if B is obtained from A by a single row operation, then we necessarily have $\det(B) = \det(A)$.

(8) ( ) If A is a 5×5 matrix, then $\det(-A) = -\det(A)$.

(9) ( ) If $\{u, v\}$ is linearly independent, then $\{u, v, u + 2v\}$ k is also linearly independent.

(10) ( ) For $x = (x_1, x_2), y = (y_1, y_2)$, $\langle x, y\rangle = x_1y_1 + 3x_2y_2$ defines an inner product on $\mathbb{R}^2$.

(11) ( ) Every linear transformation is one-to-one.

(12) ( ) Every finite dimensional inner product space has an orthogonal basis.

(13) ( ) If A is an $n \times n$ matrix, then A has always n eigenvalues.

(14) ( ) If the characteristic polynomial of a 3×3 matrix A is $\lambda^3 - 2\lambda^2 + \lambda$, then A must be invertible.

(15) ( ) If the characteristic polynomial of a 3×3 matrix A is $\lambda(\lambda - 2)(\lambda - 3)$, then A is diagonalizable.

Question 2(10 pts.) Consider the linear system

$$\begin{aligned} x_1 + x_2 + 2x_3 &= 4 \\ 2x_1 - x_2 + 4x_3 &= -3 \\ x_1 - 2x_2 + 3x_3 &= 3 \\ x_1 + x_2 + x_3 &= 6. \end{aligned}$$

(a) Find the reduced row-echelon form of the augmented matrix.

(b) Solve the linear system, if the solution exists. Check your answer.

Question 3(12 pts.) (a) Let $v_1 = (1, 2, -1), v_2 = (1, 1, 2),\ v_3 = (2, 0, 1)$.
(a) Show that $\{v_1, v_2, v_3\}$ is a basis of $\mathbb{R}^3$.

(b) Write $w = (4, 1, 3)$ as a linear combination of v_1, v_2, v_3.

Question 4(15 pts.) Let

$$A = \begin{bmatrix} 1 & 2-k & 1 & 3+k \\ 2 & 4-2k & 3 & 6+2k \\ -1 & 0 & -1 & -3-k \\ 1 & 2-k & 1 & 6+2k \end{bmatrix}$$

(a) Find the values of k for which the matrix A is invertible.

(b) For $k = 2$, find the dimension of the vector space $W = \{X \in \mathbb{R}^4 \;:\; AX = 0\}$. (Here, elements of $\mathbb{R}^4$ are considered as 4×1 matrices.)

Question 5 (10 pts.) Consider the subspace $W = \{ (a, b, c) \mid a + b - c = 0 \}$ of $\mathbb{R}^3$.

(a) Find an orthonormal basis of W with respect to the standard inner product .

(b) Find an orthonormal basis of $W^{\perp}$ with respect to the standard inner product.

(c) For $v = (1, 1, 1) \in \mathbb{R}^3$, find a vector w_1 in W and w_2 in $W^{\perp}$ such that $v = w_1 + w_2$. What is the projection $\mathrm{proj}_W(v)$ of v into W?

Question 6 (12 pts.) Let $\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 7 & -5 \\ 0 & 8 & -6 \end{bmatrix}$.

(a) Find the eigenvalues and the eigenvectors of A.

(b) If possible, find a matrix P such that $P^{-1}AP$ is diagonal. Check your answer.