M E T U
Northern Cyprus Campus

	Calculus With Analytic Geometry Short Exam 3	
Code : *Math 119* Acad.Year: *2012-2013* Semester : *Fall* Date : *03.01.2013* Time : *17:45* Duration : *45 minutes*	Last Name: Name: Student No: Signature:	
	5 QUESTIONS ON 2 PAGES TOTAL 42+2=44 POINTS	
1 | 2 | 3 | 4 | 5	KEY	

Show your work! No calculators! Please draw a box around your answers!
Please do not write on your desk!

1. (8 pts.) Evaluate the indefinite integral $\int 2013 \sin(x)\cos(3x)\,dx$.

$\sin(x+3x) = \sin x\cos 3x + \sin 3x\cos x$
$+\ \sin(x-3x) = \sin x\cos 3x - \sin 3x\cos x$
$\sin 4x + \underbrace{\sin(-2x)}_{-\sin 2x} = 2\sin x\cos 3x$

$$\frac{2013}{2}\int (\sin 4x - \sin 2x)\,dx$$

$$= \frac{2013}{2}\left(-\frac{\cos 4x}{4} - -\frac{\cos 2x}{2}\right) + C$$

$$= \boxed{\frac{2013}{8}(2\cos 2x - \cos 4x) + C}$$

2. (8 pts.) Use integration by parts to evaluate $\int_1^9 \sqrt{t}\ln t\,dt$. $= I$

$u = \ln t$	$dv = t^{1/2}dt$
$du = \frac{dt}{t}$	$v = \frac{2}{3}t^{3/2}$

$$I = \frac{2}{3}\left(t^{3/2}\ln t\right)\Big|_1^9 - \int_1^9 \frac{2}{3}t^{3/2}\cdot\frac{dt}{t}$$

$$= \frac{2}{3}\left[\left(9^{3/2}\ln(9) - 1^{3/2}\underbrace{\ln(1)}_{0}\right) - \int_1^9 t^{1/2}dt\right] = \frac{2}{3}\left[27\ln(9) - \frac{2}{3}\left(t^{3/2}\right)\Big|_1^9\right]$$

$$= 18\ln(9) - \left(\frac{2^2}{3^2}\cdot\left(9^{3/2} - 1^{3/2}\right)\right) = \boxed{18\ln(9) - \frac{104}{9}}$$

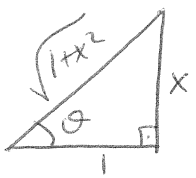
3. (8 pts.) Evaluate the indefinite integral $\int \frac{1}{(1+x^2)^2}\,dx = I$

$x = \tan\theta$

$dx = \sec^2\theta\, d\theta$

$$I = \int \frac{1}{(\sec^2\theta)^2}\sec^2\theta\, d\theta = \int \frac{d\theta}{\sec^2\theta} = \int \cos^2\theta\, d\theta = \frac{1}{2}\int (1+\cos 2\theta)\,d\theta$$

$$= \frac{\theta}{2} + \frac{\sin 2\theta}{4} + C$$



$$= \frac{\arctan x}{2} + \frac{1}{4}(2\sin\theta\cos\theta) + C = \boxed{\frac{\arctan x}{2} + \frac{x}{2(1+x^2)} + C}$$

4. ($3 \times 4 = 12$ pts.) For each of the indefinite integrals below, write the trigonometric substitution that would be the most helpful in evaluating the integral.

DO NOT EVALUATE THE INTEGRALS.

(a) $\int x^2\sqrt{16+x^2}\,dx$

$\boxed{x = 4\tan\theta}$

(b) $\int \frac{x^2\,dx}{\sqrt{16-x^2}}$

$\boxed{x = 4\sin\theta}$

(c) $\int \sqrt{x^2-16}\,dx$

$\boxed{x = 4\sec\theta}$

5. (8 pts.) What is the correct form of the partial fraction decomposition of the following?

DO NOT CALCULATE THE COEFFICIENTS.

$$\frac{x^4}{(x^3+x)^2(x^2-x+3)}$$

1st Soln.

$$\frac{A_1}{x} + \frac{A_2}{x^2} + \frac{B_1x+C_1}{x^2+1} + \frac{B_2x+C_2}{(x^2+1)^2} + \frac{Dx+E}{x^2-x+3}$$

2nd Soln.

$$\frac{A_1x+B_1}{x^2+1} + \frac{A_2x+B_2}{(x^2+1)^2} + \frac{Cx+D}{x^2-x+3}$$