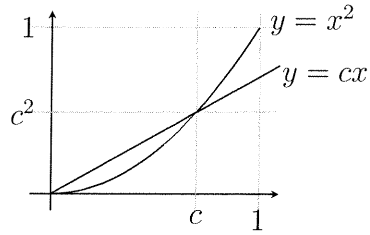
	Calculus and Analytical Geometry II. Midterm	
Code : *Math 119* Acad.Year: *2010-2011* Semester : *Fall* Date : *18.12.2010* Time : *10:00* Duration : *120 minutes*	Last Name: Name : Department: Signature:	Student No: Section:
	6 QUESTIONS ON 6 PAGES TOTAL 100 POINTS	

1	2	3	4	5	6		

1. (15 pts) Find c so that the area between the curves $y = x^2$ and $y = cx$ for $0 \le x \le 1$ is minimized (see the graph).



For $0 \leq t \leq 1$, the area between $y=x^2$ and $y=tx$ on $[0,1]$ is:

$$f(t) = \int_0^t (tx - x^2)\,dx + \int_t^1 (x^2 - tx)\,dx$$

$$f(t) = \frac{tx^2}{2} - \frac{x^3}{3}\Big|_0^t + \frac{x^3}{3} - \frac{tx^2}{2}\Big|_t^1$$

$$f(t) = \frac{t^3}{3} - \frac{t}{2} + \frac{1}{3}$$

$f'(t) = t^2 - \frac{1}{2}$, so $t = \frac{1}{\sqrt{2}}$ is the only crit. point on $[0,1]$.

$f'(t) < 0$ on $(0, \frac{1}{\sqrt{2}})$ and $f'(t) > 0$ on $(\frac{1}{\sqrt{2}}, 1)$

So $t = \frac{1}{\sqrt{2}}$ is a global min on $[0,1]$.

Alternatively $f\left(\frac{1}{\sqrt{2}}\right) = \frac{2\sqrt{2} - 2}{6\sqrt{2}} < f(0), f(1)$

So is a global min of $f(t)$ on $[0,1]$

② Sketch the graph of $f(x) = \frac{x^2-4}{(x+1)^2}$

a) Domain : $\mathbb{R} \setminus \{-1\}$

b) y-intercept -4

x-intercepts $-2, 2$

c) $\lim_{x \to -1^-} f(x) = -\infty$ $\lim_{x \to -1^+} f(x) = -\infty$, $x = -1$ vert. asym

$\lim_{x \to \infty} f(x) = 1 = \lim_{x \to -\infty} f(x)$, $y = 1$ hor. asymp.

d) $f'(x) = \frac{2x(x+1)^2 - (x^2-4)2(x+1)}{(x+1)^4} = \frac{2(x+1)(x^2+x-x^2+4)}{(x+1)^4}$

$f'(x) = \frac{2(x+1)(x+4)}{(x+1)^4}$, $x = -4$ crit point

	-4	-1	
$f'(x)$	$+$	$-$	$+$
$f(x)$	Inc.	Dec	Inc.

e) $x = -4$ is a local max point

② (Contined)

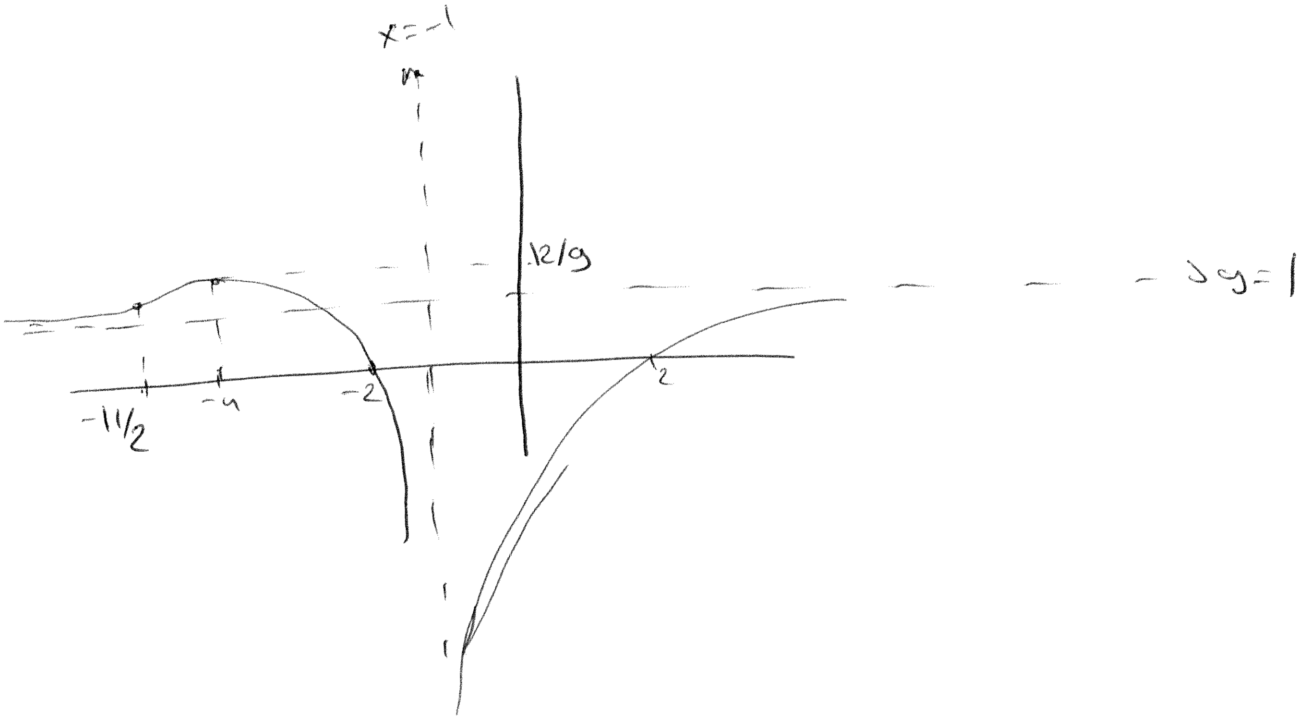
f) $$f''(x) = \frac{2(2x+5)(x+1)^4 - 2(x+1)(x+4)\cdot 4\cdot(x+1)^3}{(x+1)^8}$$

$$f''(x) = \frac{2(x+1)^4(2x+5-4x-16)}{(x+1)^8} = \frac{2(x+1)^4(-2x-11)}{(x+1)^8}$$

$f''(x) = 0$ when $x = -\frac{11}{2}$

		$-11/2$		-1	
$f''(x)$	$+$	0	$-$		$-$
$f(x)$	Conc. Up		Conc. D.		Conc. D.

g)

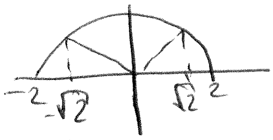


3. (20 pts) Compute the following integrals.

(a) $\int_{-\sqrt{2}}^{\sqrt{2}} \sqrt{4-x^2}\, dx$ (Hint: Interpret the integral as area.)

$y = \sqrt{4-x^2}$ is the semi-circle centered at the origin with radius 2

So $\int_{-\sqrt{2}}^{\sqrt{2}} \sqrt{4-x^2} = \frac{\pi . (2^2)}{4} + 2\left(\frac{\sqrt{2} . \sqrt{2}}{2}\right) = \pi + 2$



(b) $\int \cos x \sec^2(\sin x)\, dx = \int \sec^2 u . du = \tan u + C$

$= \tan(\sin x) + C$

$u = \sin x$

$du = \cos x\, dx$

(c) $\int 4x^3\sqrt{x^2+1}\, dx = 2\int (u-1)\sqrt{u} . du = 2\int \left(u^{3/2} - u^{1/2}\right) du$

$u = x^2+1 \Rightarrow x^2 = u-1$

$du = 2x . dx$

$= 2 . \frac{2}{5} u^{5/2} - 2 . \frac{2}{3} u^{3/2} + C$

$= \frac{4}{5}(x^2+1)^{5/2} - \frac{4}{3}(x^2+1)^{3/2} + C$

(d) $\int (\sqrt[3]{x} - \frac{1}{\sqrt[3]{x}})(x + \sqrt[3]{x})\, dx = \int \left(x^{4/3} + x^{2/3} - x^{2/3} - 1\right) dx$

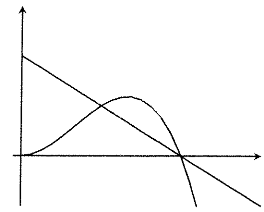
$= \frac{3}{7} x^{7/3} - x + C$

(e) $\int_{-1}^{1} x \left|\tan x - \sin x\right|\, dx$

$f(x) = x\,|\tan x - \sin x|$ is an odd function. $(f(-x) = -f(x))$

So $\int_{-1}^{1} f(x)\, dx = 0$

4. (20 pts) In the following parts, write but DO NOT SOLVE integrals computing the volume given by rotating the region between $y = x^2(2-x)$ and $y = 2-x$ around the following:



(a) the x-axis.

Intersection points $x^2(2-x) = (2-x)$

$x=1$ & $x=2$

$$V = \int_1^2 \pi \left([x^2(2-x)]^2 - (2-x)^2 \right) dx$$

(b) the y-axis.

$$V = \int_1^2 2\pi x \left(x^2(2-x) - (2-x) \right) dx$$

(c) the line $x = -1$.

$$V = \int_1^2 2\pi (x+1) \left(x^2(2-x) - (2-x) \right) dx$$

(d) the line $y = 3$.

$$V = \int_1^2 \pi \left([3-(2-x)]^2 - [3 - x^2(2-x)]^2 \right) dx$$

5. (15 pts) This problem has two unrelated parts.

(a) $F(x) = \int_x^{2x} f(t)\,dt$ and $f(t) = \int_t^{t^2} u^2 \tan u\,du$.

Find $F''(x)$.

$$f(2x).2 - f(x)$$

$$F(x) = \int_0^{2x} f(t)\,dt - \int_0^{x} f(t)\,dt \quad , \quad F'(x) =$$

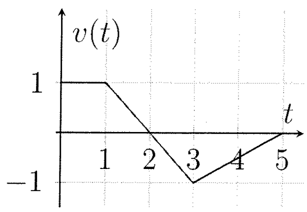
$$f(t) = \int_0^{t^2} u^2 \tan u \,.du - \int_0^{t} u^2 \tan u \,.du$$

$$f'(t) = (t^4 \tan t^2)(2t) - t^2.\tan t$$

So $F''(x) = 4.f'(2x) - f'(x) =$

$$4(16x^4 \tan 4x^2)4x - 4(4x^2 \tan 2x) - (x^4 \tan x^2)2x$$
$$- x^2.\tan x$$

(b) An object moves along a straight line with velocity given by the graph to the right.



(i) What is the change in position at time $t = 3$?

$$\int_0^3 v(t)\,dt = 1 + \frac{1}{2} - \frac{1}{2} = 1$$

(ii) What is the total distance traveled at time $t = 5$? (Hint: It is not 0.)

$$\int_0^5 |v(t)|\,dt = 1 + \frac{1}{2} + \frac{1}{2} + 1 = 3$$

6. (10 pts) Let f be a <u>nonlinear</u> function, twice-differentiable on $[0, 2]$ with $f(0) = 0$, $f(1) = 2$, and $f(2) = 4$.

(a) Show that $f'(x) = 2$ for at least two different values of x between 0 and 2.

By M.V.T., there is $c \in (0,1)$ s.t. $f'(c) = \dfrac{f(1) - f(0)}{1 - 0} = 2$

By M.V.T., there is $c \in (1,2)$ s.t. $f'(c) = \dfrac{f(2) - f(1)}{2 - 1} = 2$

(b) Show that $f''(x) = 0$ somewhere between 0 and 2.

By part (a), there is $c \in (0,1)$ s.t. $f'(c) = 2$

and there is $d \in (1,2)$ s.t. $f'(d) = 2$.

Then by MVT applied to $f'(x)$:

there is $e \in (0,2)$ s.t. $f''(e) = \dfrac{f'(d) - f'(c)}{d - c} = 0$