METU - NCC

	Precalculus Midterm	
Code : *Math 100* Acad.Year: *2011-2012* Semester : *Spring* Date : *18.4.2012* Time : *17:40* Duration : *80 minutes*	Last Name: Name : Department: Signature:	Student No.: Section:
	12 QUESTIONS ON 6 PAGES TOTAL 100 POINTS	

1	(8)	2	(7)	3	(8)	4	(7)	5	(14)	6	(7)	7	(7)	8	(7)	9	(7)	10	(14)	11	(7)	12	(7)	

1. *(8 pts)* Find all x satisfying $|100x + 20| > -120 - x$

Say: $100x+20 \geq 0 \Rightarrow x \geq \frac{-1}{5}$ So: $100x+20 > -120-x \Leftrightarrow 101x > -140 \Leftrightarrow x > \frac{-140}{101}$

Solution Set: $(-\infty, \frac{-1}{5}] \cup [\frac{-1}{5}, +\infty)$

Say: $100x+20 \leq 0 \Rightarrow x \leq \frac{-1}{5}$. So: $-100x-20 > -120-x \Leftrightarrow 99x < 100 \Leftrightarrow x < \frac{100}{99}$

2. *(7 pts)* A mothers age is 3 times the sum of her two children. After 26 years, mothers age will be equal to the sum of her two children. How old was the **mother** when she had her first child if her son is 7 years older than her daughter?

mothers age: m
sons age: $s = d+7$ (*)
daughters age: d

To find: $m - s = 39 - 10 = 29$

29

$m = 3(s+d)$

$m+26 = s+26+d+26$ (*)

$m = 3.(s+d) = s+d+26 \Rightarrow s+d = 13 \overset{(*)}{=} 2s-7 \Rightarrow s = 10$

$m = 3.13 = 39$

3. *(8 pts)* Find all values of x satisfying $3x^2 + 3x - 40 = 9x + 65$.

$3x^2+3x-40 = 9x+65 \Leftrightarrow 3x^2-6x-105 = 0$

$\Leftrightarrow x^2-2x-35 = 0 \Leftrightarrow (x-7).(x+5) = 0.$

Solution Set $= \{7, -5\}$

4. *(7 pts)* Simplify $\dfrac{(1+i)(1+2i)(1+3i)}{(\frac{1}{i}+1)(\frac{1}{i}-1)(\frac{2}{i}+i)(\frac{2}{i}-i)}$

Remember: $(x-y)\cdot(x+y) = x^2 - y^2$.

$\left(\frac{1}{i}+1\right)\cdot\left(\frac{1}{i}-1\right) = \frac{1}{-1} - 1 = -2.$

$\left(\frac{2}{i}+i\right)\cdot\left(\frac{2}{i}-i\right) = \frac{4}{-1} - (-1) = -4+1 = -3$

$(1+i)\cdot(1+2i)\cdot(1+3i) = (1-2+3i)\cdot(1+3i)$

$= (3i-1)\cdot(3i+1) = -9-1 = -10.$

So: $\dfrac{(1+i)\cdot(1+2i)(1+3i)}{\left(\frac{1}{i}+1\right)\cdot\left(\frac{1}{i}-1\right)\cdot\left(\frac{2}{i}+i\right)\left(\frac{2}{i}-i\right)} = \dfrac{-10}{(-2)\cdot(-3)} = \dfrac{-5}{3}$

$-5/3$

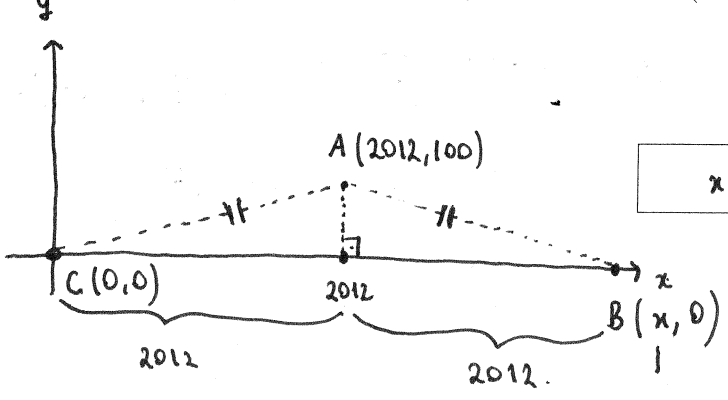
5. *(2x7 pts)* Find α and β so that $x^2 + x + 2 = (x+\alpha)^2 + \beta$.

$x^2 + x + 2 = x^2 + x + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 2$

$= \left(x + \frac{1}{2}\right)^2 + \frac{7}{4}$

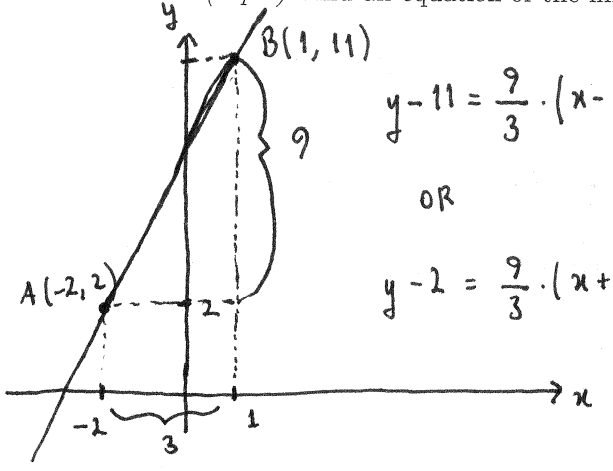
$\alpha = 1/2$ $\qquad$ $\beta = 7/4$

6. *(7 pts)* Find the x-coordinate of the point B$=(x,0)$ so that AB=AC; where A $= (2012, 100)$ and C$= (0,0)$. That is the triangle ABC is isoceles(ikizkenar).



$x = 2.2012 = 4024$

7. *(7 pts)* Find an equation of the line passing through A(-2,2) and B(1,11).



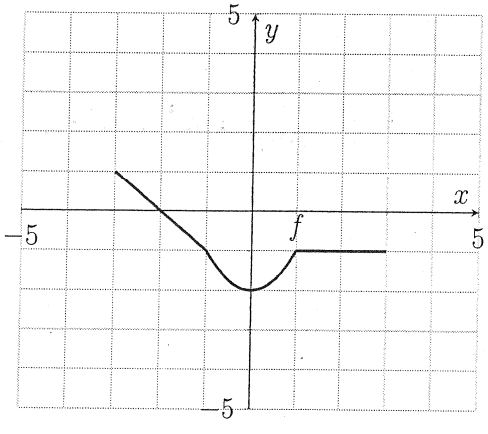
$y - 11 = \frac{9}{3} \cdot (x - 1) \Leftrightarrow y = 3x + 8$

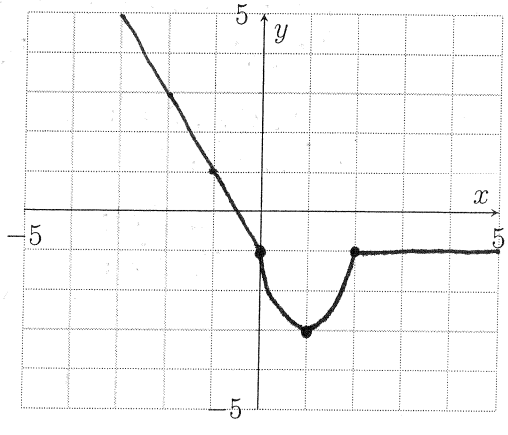
OR

$y - 2 = \frac{9}{3} \cdot (x + 2) \Leftrightarrow y = 3x + 8$

$\boxed{y = 3x + 8}$

8. *(7 pts)* The graph of $f(x)$ is given below. Graph $g(x) = 2f(x - 1) + 1$ in the blank grid provided.





9. *(7 pts)* Find the domain of the function $f(x) = \dfrac{\sqrt{1 - x^2}}{(x^3 - 8)}$.

$1 - x^2 \geq 0 \Leftrightarrow x^2 \leq 1 \Leftrightarrow |x| \leq 1 \Leftrightarrow x \in [-1, 1]$.

and

$x^3 - 8 \neq 0 \Rightarrow x \neq 2$

$\boxed{\text{Domain}(f) = [-1, 1]}$

10. *(2x7 pts)* Complete the table below by filling in correct values of $(f \circ g)(x)$ and $(f \circ g^{-1})(x)$:

x	1	2	3	4
$f(x)$	4	1	2	3
$g(x)$	2	4	1	3
$(g \circ f)(x)$	$g(4)=3$	$g(1)=2$	$g(2)=4$	$g(3)=1$
$(g \circ f^{-1})(x)$	$g(2)=4$	$g(3)=1$	$g(4)=3$	$g(1)=2$

11. *(7 pts)* Divide $P(x) = x^6 - 4x^4 + 2x^2 + 4$ by $Q(x) = x^2 - 2$.

$$
\begin{array}{r|l}
x^6 - 4x^4 + 2x^2 + 4 & x^2 - 2 \\
\cline{2-2}
-\,x^6 \mp 2x^4 & x^4 - 2x^2 - 2 \\
\cline{1-1}
\quad -2x^4 + 2x^2 + 4 & \\
-\,\mp 2x^4 \mp 4x^2 & \\
\cline{1-1}
\quad\quad -2x^2 + 4 & \\
-\,\mp 2x^2 \mp 4 & \\
\cline{1-1}
\quad\quad\quad 0 &
\end{array}
$$

$P(x)/Q(x) = x^4 - 2x^2 - 2$

12. *(7 pts)*Describe a function $g(x)$ in terms of $f(x)$ if the graph of g is obtained by shifting the graph of f to the left by 2 units and up by 4 units.

$g(x) = f(x+2) + 4$