| | Calculus and Analytical Geometry<br>I. Midterm | |
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| Code : *Math 119*<br>Acad.Year : *2010-2011*<br>Semester : *Spring*<br>Date : *26.3.2011*<br>Time : *13:30*<br>Duration : *120 minutes* | Last Name:<br>Name :<br>Department:<br>Signature:<br>Student No:<br>Section: | |
| | 8 QUESTIONS ON 8 PAGES<br>TOTAL 100 POINTS | |

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | | |
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**Show your work! Please draw a box around your answers!**

**1.**(5 pts) Let $f(x) = \begin{cases} x\big(1 + x\cos\frac{1}{x}\big), & \text{if } x \neq 0 \\ c, & \text{if } x = 0. \end{cases}$

What value of $c$ will make $f$ continuous?

**2.** (5+5+5+5 pts) Evaluate the following limits, if they exist. Show your work. **Do not use L'Hospital's rule.**

(a) $\lim_{x\to\infty}\left(2x-\sqrt{4x^2-3x}\right)$.

(b) $\lim_{x\to 0}\dfrac{\sqrt{1-\cos x}}{1-x}$.

(c) $\lim_{x\to 4}\dfrac{x-4}{2-\sqrt{x}}$.

(d) $\lim_{x\to 0}\dfrac{(\tan x)^2+\sin(x^3)}{2x^2}$.

**3.** (5+5 pts) The following questions deal with definitions of the derivative.

(a) The function $y = f(x)$ has secant lines through the points $\big(2, f(2)\big)$ and $\big(a, f(a)\big)$ given by

$$y = \left(\frac{a^2 - 3a + 2}{a - 2}\right)(x - 2) + 5$$

for every $a$. What is the derivative $f'(2)$?

(b) If plugging into the definition of the derivative **immediately** yields

$$g'(1) = \lim_{h \to 0} \frac{\sqrt{h^2 + 2h + 4} - 2}{h}$$

then what is $g(x)$?

**4.** (5+5+5+5 pts) Calculate the following derivatives.

(a) $\dfrac{d}{dx}\big((x^2+1)\sin x\cos x\big)$.

(b) $\dfrac{d}{dx}\ \sec\big(\tan x+\sqrt{\sin x}\big)$.

(c) Find $y'$ in terms of $x$ and $y$ if $2\cos x\sin y=3$.

(d) Find $y''$ in terms of $x$ and $y$ if $2\cos x\sin y=3$.

**5.** (10 pts) Find the equation of the tangent line to the curve

$$y^2 + \sin(xy) - x^2 = \frac{\pi^2}{4}$$

at the point $(1, \frac{\pi}{2})$.

**6.** (10 pts) A ball is being filled with air at a rate of 3 $\frac{\text{cm}^3}{\text{s}}$. At what rate is the surface area changing when the ball has surface area 36 $\text{cm}^2$?

(Remember that SA $= 4\pi r^2$, and Vol $= \frac{4}{3}\pi r^3$.)

**7.** (15 pts) Find the absolute minimum and absolute maximum of the function

$$f(x) = 4\sqrt{|x|} - x^2 + 1$$

on the interval $[-2, 2]$.

**8.** (10 pts) Use *linear approximation* to estimate the value of $(27.01)^{\frac{4}{3}}$.