Math 219
Fall 2010
Homework II

Due date: October 22th, 2010

1. Consider the three equations

   (a) $(y^2 - x) + xy\frac{dy}{dx} = 0$,

   (b) $(y^2 - x^3) + (2xy + y^3)\frac{dy}{dx} = 0$,

   (c) $y + (2x - ye^y)\frac{dy}{dx} = 0$.

   One of these equations is exact, one has an integrating factor depending only on x and one with an integrating factor depending on y only. Find these equations and solve them.

2. Solve the ode $(-xy\sin x + 2y\cos x)dx + 2x\cos x dy = 0$ given that $\mu(x, y) = xy$ is an integration factor.

3. Consider the linear homogeneous differential equation

$$(1 + x)y'' - (2 + x)y' + y = 0.$$

   (a) Verify that $y_1(x) = e^x$ and $y_2(x) = x + 2$ for $x \in (0, \infty)$ are solutions of the differential equation. By calculating the Wronskian y_1 and y_2 show that $\{y_1, y_2\}$ is a fundamental set of solutions.

   (b) Find the solution of the differential equation satisfying $y(0) = 3$ and $y'(0) = 1$.

   (c) Find the largest possible interval in which the solution to the problem described in (b) exists and unique.

4. Find A such that the differential equation

$$(x^3 + 3xy)dx + (Ax^2 + 4y)dy = 0$$

   is an exact equation.

   (a) Find the solution of the resulting equation that passes through the point $(x, y) = (0, 1)$.

   (b) Write down the explicit solution found in (b).

5. Consider the initial value problem

$$y'' + 2y' + 6y = 0, \qquad y(0) = 2, \quad y'(0) = a \geq 0.$$

   (a) Find the solution $y(t)$ of this problem.

   (b) Find a so that $y = 0$ when $t = 1$.