Math 219
Fall 2010
Homework I

Due date: October 8th, 2010

1. Find an explicit solution of the initial value problem

$$\frac{dy}{dx} = \frac{2y + \sqrt{x^2 - y^2}}{2x}, \quad y(1) = 0.$$

2. Solve the initial value problem

$$\frac{dy}{dx} = \frac{xy + 2y^2}{xy + x^2}, \quad y(1) = 1.$$

3. Solve the differential equation

$$\frac{dy}{dx} + \frac{2}{x}y = x.$$

4. Find the solution of the initial value problem

$$ty' - 2y = t^4 \cos(t^2), \quad y((\sqrt{\pi}) = 1.$$

5. Draw direction fields, determine the equilibrium points, determine their stabilities of the following autonomous differential equations

   (a) $y' = y^2 - 9y$.
   (b) $y' = y^2 - 5y + 6$.
   (c) $y' = -y^2 + 6y - 9$.

6. Use the substitution $v = \frac{1}{y}$ to solve $xy' - (1 + 2x)y = y^2, \quad x > 0$.

7. Solve the differential equation $xy' + y = x^3y^6$.

8. Suppose that a tank of volume 100 liters initially contains 50 liters of pure water. A salt water solution of 1 gram/liter is incoming to the tank at a rate of 3 liters/min, and at the same time the mixture is leaving the tank from a hole at the bottom at a rate of 1 liter/min. The mixture inside the tank is continuously stirred.

   (a) Express the volume of water in the tank at time t in terms of t.
   (b) Write a differential equation for the amount $Q(t)$ of salt in the tank at time t. (Hint: rate of salt going out will depend on both Q and t).
   (c) Solve this equation.
   (d) Find the amount of salt in the tank when the tank fills up.