Math 120

Fall 2010

Homework I

Due date: October 22th, 2010

1. Determine whether the following sequences converge or diverge. Find the limits of the ones which converge.

   (a) $a_{n+1} = \frac{1}{2}(1 + a_n^2)$, $\quad a_1 = 0$.

   (b) $a_{n+1} = \frac{2 + \sin n}{\sqrt{n}}$.

   (c) $a_{n+1} = \ln\left(\frac{n}{n^2 + 1}\right)$.

   (d) $a_{n+1} = \left(\frac{n}{n+1}\right)^n$.

2. Find the sum of the series

$$\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt[3]{n}} - \frac{1}{\sqrt[3]{n+1}} \right).$$

3. Determine whether the following series converge or diverge. Find the limits of the ones which converge.

   (a)
$$\sum_{n=1}^{\infty} \left(\frac{\ln n}{\ln(n^2 + 1)} \right)^n.$$

   (b)
$$\sum_{n=1}^{\infty} \frac{\sqrt{n^2 + 2n}}{n^4 - 3n^2 - 1}.$$

   (c)
$$\sum_{n=1}^{\infty} \frac{2 \cdot 4 \cdot 6 \cdots 2n}{n! \cdot 3^n}.$$

   (d)
$$\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{n + 10}.$$

   (e)
$$\sum_{n=1}^{\infty} (-1)^n \frac{2n + 1}{3n + 2}.$$