1. (10 pts) Find all solutions of $x$ satisfying $||x-1|-1| \geq 1$.

\[ \begin{align*}
||x-1|-1| \geq 1 & \implies |x-1|-1 \geq 1 \quad \text{or} \quad |x-1|-1 \leq -1 \\
& \implies |x-1| \geq 2 \quad \text{or} \quad |x-1| \leq 0 \\
& \implies x \leq -\frac{1}{2} \quad \text{or} \quad x \geq 2 \\
& \implies x \geq 3 \quad \text{or} \quad x \leq -1 \quad \text{or} \quad x = 1
\end{align*} \]

\[ (-\infty, -1] \cup \{1\} \cup [3, \infty) \]

2. (15 pts) Find all solutions of $x$ satisfying $(x^2 - x)^2 - 8(x^2 - x) + 12 = 0$.

Say $x^2 - x = a$, eqn becomes, $a^2 - 8a + 12 = 0$

\[ (a-2)(a-6) = 0 \implies a = 2 \quad \text{or} \quad a = 6 \]

When $a = 2$: $x^2 - x = 2 \implies x^2 - x - 2 = 0 \implies (x-2)(x+1) = 0 \implies x = -1 \quad \text{or} \quad x = 2$

When $a = 6$: $x^2 - x = 6 \implies x^2 - x - 6 = 0 \implies (x-3)(x+2) = 0 \implies x = -2 \quad \text{or} \quad x = 3$

\[ \text{Solutions: } -2, -1, 2, 3 \]
3. (10 pts) Give an example to a fifth degree polynomial $P(x)$ such that $P(0) = P(1) = P(2) = 3$.

$$P(x) = Q(x)(x-0)(x-1)(x-2) + 3 \quad \text{where } Q(x) \text{ is any quadratic function.}$$

Specifically,

$$P(x) = (ax^2 + bx + c)x(x-1)(x-2) + 3$$

$$P(x) = (ax^2 + bx + c)(x-0)(x-1)(x-2) + 3$$

4. (10 pts) Use synthetic division to calculate $P(7)$ where $P(x) = x^4 - 2x^2 - 5x + 3$.

\[1\ 0\ -2\ -5\ 3\]

\[+\quad 7\ 49\ 329\ 2268\]

\[7\ 1\ 7\ 47\ 324\ 2271\]

$P(7) = 2271$

5. (10 pts) Find the domain of the function $f(x) = \frac{1}{|x+1| - \sqrt{4x}}$.

To define $\sqrt{4x}$, $4x \geq 0 \Rightarrow x \geq 0$.

To define $f(x)$, denominator must be non-zero;

\[|x+1| - \sqrt{4x} \neq 0 \Rightarrow |x+1| \neq \sqrt{4x}\]

\[\Rightarrow x^2 + 2x + 1 \neq 4x \Rightarrow x^2 - 2x + 1 \neq 0\]

\[\Rightarrow (x-1)^2 \neq 0 \Rightarrow x \neq 1\]

Domain $f(x) = \left[0, \infty\right) - \{1\}$
6. (15 pts) What is the equation of the line which passes through the vertex of the parabola
\[ y = ax^2 - 12x + 8 \text{ and the origin.} \]
Let's write our function in vertex form:
\[ 6x^2 - 12x + 8 = 6(x^2 - 2x + 1 - 1) + 8 = 6(x-1)^2 + 2. \]
So the vertex is \((1,2)\).
Line eqn passing through \((1,2)\) and \((0,0)\) is:
\[ y = \frac{2}{1}x. \]

7. (15 pts) Find an equation of the circle passing through the points \(A = (16,0)\) and \(B = (0,0)\).

Radius of the circle is:
\[ \sqrt{(8-0)^2 + (6-0)^2} = 10. \]
Eqn of the circle is:
\[ (x-8)^2 + (y-6)^2 = 10^2. \]
8. (15 pts) Graph of the function $f(x)$ is given below. Using basic transformations sketch the graph of $g(x) = 2 - f(-x + 1)$ in the blank grid. Show individual steps of the transformation of the graph of $f(x)$ to the graph of $g(x)$. 

\[\begin{align*}
\text{Graph of } f(x) & \quad & \text{Graph of } -f(x) \\
\text{Graph of } -f(x+1) & \quad & \text{Graph of } -f(-x+1) \\
\text{Graph of } 2-f(-x+1) & \quad & \\
\end{align*}\]