METU - NCC

	Precalculus Midterm	
Code : *Math 100* Acad.Year: *2013-2014* Semester : *Spring* Date : *14.4.2012* Time : *18:40* Duration : *80 minutes*	Last Name: Name : Department: Signature:	Student No.: Section:
	8 QUESTIONS ON 4 PAGES TOTAL 100 POINTS	

1	2	3	4	5	6	7	8	
(10)	(15)	(10)	(10)	(10)	(15)	(15)	(15)	

1. *(10 pts)* Find all solutions of x satisfying $||x-1|-1| \geq 1$.

$||x-1|-1| \geq 1 \Rightarrow |x-1|-1 \geq 1$ or $|x-1|-1 \leq -1$

$\Rightarrow |x-1| \geq 2$ or $|x-1| \leq 0$

$\Rightarrow x-1 \geq 2$ or $x-1 \leq -2$ or $x-1=0$

$\Rightarrow x \geq 3$ or $x \leq -1$ or $x=1$

$(-\infty,-1] \cup \{1\} \cup [3,\infty)$

2. *(15 pts)* Find all solutions of x satisfying $(x^2-x)^2 - 8(x^2-x) + 12 = 0$.

Say $x^2-x=a$, eqn becomes, $a^2-8a+12=0$

$(a-2)(a-6)=0 \Rightarrow a=2$ or $a=6$

when $a=2$; $x^2-x=2 \Rightarrow x^2-x-2=0 \Rightarrow (x-2)(x+1)=0 \Rightarrow x=-1$ or $x=2$

when $a=6$; $x^2-x=6 \Rightarrow x^2-x-6=0 \Rightarrow (x-3)(x+2)=0 \Rightarrow x=-2$ or $x=3$

Solutions: $-2,-1,2,3$

3. *(10 pts)* Give an example to a fifth degree polynomial $P(x)$ such that $P(0) = P(1) = P(2) = 3$.

$P(x) = Q(x)(x-0)(x-1)(x-2)+3$ where $Q(x)$ is any quadratic function.

Specifically,

$P(x) = (ax^2+bx+c) \times (x-1)(x-2)+3$

$\boxed{P(x)=(ax^2+bx+c)(x-0)(x-1)(x-2)+3}$

4. *(10 pts)* Use synthetic division to calculate $P(7)$ where $P(x) = x^4 - 2x^2 - 5x + 3$.

	1	0	−2	−5	3
+		7	49	329	2268
7	1	7	47	324	2271

$\boxed{P(7)=2271}$

5. *(10 pts)* Find the domain of the function $f(x) = \dfrac{1}{|x+1| - \sqrt{4x}}$.

To define $\sqrt{4x}$; $4x \geqslant 0 \Rightarrow x \geqslant 0$

To define $f(x)$, denominator must be non-zero;

$|x+1| - \sqrt{4x} \neq 0 \Rightarrow |x+1| \neq \sqrt{4x}$

$\Rightarrow x^2+2x+1 \neq 4x \Rightarrow x^2-2x+1 \neq 0$

$\Rightarrow (x-1)^2 \neq 0 \Rightarrow x \neq 1$

$\boxed{\text{Domain } f(x) = [0,\infty) - \{1\}}$

6. *(15 pts)* What is the equation of the line which passes through the vertex of the parabola $y = 6x^2 - 12x + 8$ and the origin.

Let's write our function in vertex form,

$$6x^2 - 12x + 8 = 6(x^2 - 2x + 1 - 1) + 8 = 6(x-1)^2 + 2$$

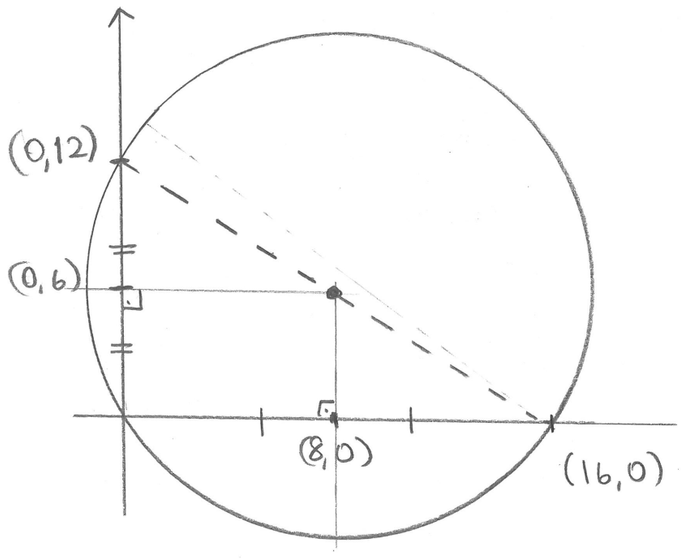
So the vertex is $(1, 2)$

Line eqn passing through $(1,2)$ and $(0,0)$ is:

$$\frac{y-0}{x-0} = \frac{2-0}{1-0} \Rightarrow y = 2x$$

$\boxed{y = 2x}$

7. *(15 pts)* Find an equation of the circle pasiing through the points $A = (16, 0)$ and $B = (0, 12)$ and $O = (0, 0)$.



Center of the circle is: $(8,6)$

Radius is: $\sqrt{(8-0)^2 + (6-0)^2} = 10$

Eqn of the circle is:

$$(x-8)^2 + (y-6)^2 = 10^2$$

$\boxed{(x-8)^2 + (y-6)^2 = 10^2}$

8. *(15 pts)* Graph of the function $f(x)$ is given below. Using basic trasformations sketch the graph of $g(x) = 2 - f(-x+1)$ in the blank grid. Show individual steps of the transformation of the graph of $f(x)$ to the graph of $g(x)$.