| Math 210 | Applied Mathematics for Engineers | II. Exam | 28.04.2010 |
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| Last Name : | Dept./Sec. : | Signature |
| Name : | Time : 17: 40 | |
| Student No: | Duration : 110 *minutes* | |

| 5 QUESTIONS ON 6 PAGES | TOTAL 100 POINTS |
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**Q1 (5+5=10 pts.)** Consider the space transformation $\mathrm{T} : \mathbb{R}^3 \to \mathbb{R}^3$ which reflects points across the $y = x$ plane in space.

**(i)** Write T as a matrix (using the standard basis).

**(ii)** Is this transformation invertible? Justify your answer.

**Q2 (10+5=15 pts.)** In the following two parts, $\mathbf{A}$, $\mathbf{w}_1, \mathbf{w}_2$, and $\mathbf{v}$ are as follows:

$$\mathbf{A} = \begin{bmatrix} 4 & 6 & -2 & -4 & -2 \\ 6 & 4 & 0 & -2 & -6 \\ 0 & 0 & 2 & 2 & -2 \\ 6 & 6 & 0 & -4 & -6 \\ 8 & 8 & -2 & -6 & -6 \end{bmatrix} \qquad \mathbf{w}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \qquad \mathbf{w}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} \qquad \mathbf{v} = \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The vector $\mathbf{v}$ is an eigenvector of $\mathbf{A}$. One of $\mathbf{w}_1$, $\mathbf{w}_2$ is an eigenvector but the other isn't.

**(i)** Which one of $\mathbf{w}_1$ and $\mathbf{w}_2$ is the eigenvector? Show your work.

**(ii)** What is the eigenvalue for the eigenvector $\mathbf{v}$? Show your work.

**Q3 (10+10+10=30 pts.)** The following questions deal with a matrix $\mathbf{A}$ which has eigenvalues $\lambda_1 = 2$ and $\lambda_2 = 1$ and $\lambda_3 = -2$ and associated eigenvectors:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \text{and} \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \quad \text{and} \quad \mathbf{v}_3 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

*(Solve these questions without computing* $\mathbf{A}$*.)*

**(i)** What is the determinant of $\mathbf{A}$?

**(ii)** Is $\mathbf{A}$ symmetric? Justify your answer.

**(iii)** What are the eigenvalues and eigenvectors of $\mathbf{A}^2$?

**(iv)** What is the solution to $\mathbf{A}\mathbf{x} = \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$?

**Q4 (5+15+10=30 pts.)** The following parts deal with computations of eigenvalues and eigenvectors.

**(i)** Find the eigenvalues of $\begin{bmatrix} 2 & -2 & 0 \\ -1 & 1 & 0 \\ -1 & 1 & 0 \end{bmatrix}$. (Do not find eigenvectors.)

**(ii)** The matrix $\begin{bmatrix} 2 & 2 & 0 \\ 1 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix}$ has eigenvalues $\lambda = 2, 2, 0$. Find associated eigenvectors.

What is unusual about your result?

**(iii)** Compute the matrix $\mathbf{A}$ with eigenvalues $\lambda_1 = 2$ and $\lambda_2 = 0$ and $\lambda_3 = 1$ and associated eigenvectors

$$\mathbf{v}_1 = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\1\end{bmatrix} \quad \text{and} \quad \mathbf{v}_2 = \begin{bmatrix}0\\1\\0\end{bmatrix} \quad \text{and} \quad \mathbf{v}_3 = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\-1\end{bmatrix}.$$

(Write your answer as a matrix – **not** as a product of matrices.)

**Q5 (5+10=15 pts.)** $\mathbf{A}$ is a real, symmetric matrix with eigenvectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \quad \text{and} \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \quad \text{and} \quad \mathbf{v}_3 = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}.$$

**(i)** Are the vectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ all orthogonal to each other?

**(ii)** What can you say about the eigenvalues associated to $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$?
*(Note: There is no mistake in this problem: There are real, symmetric matrices with these eigenvectors.)*